CALCULO DE UNA POLIGONAL ELECTRÓNICA

A. CALCULO PRELIMINAR

1) Dibujo de la poligonal a escala.

2) Calculo de la superficie de la poligonal.

En forma grafica

- En forma gráfica.
- Con planímetro.
- Por coordenadas.
- Con papel milimetrado.

3) Encontrar una latitud que pueda ser identificada como latitud media en la poligonal.

4) Calculo de los radios de curvatura: $\rho_m = \dfrac{a \cdot (1 - e^2)}{(1 - e^2 \cdot sen^2 \varphi_m)^{3/2}}$; $N_m = \dfrac{a}{(1 - e^2 \cdot sen^2 \varphi_m)^{1/2}}$

5) Calculo del exceso esférico: $E'' = \dfrac{Superficie}{N \cdot \rho \cdot arc1''}$

6) Cierre angular de la poligonal, quedando los ángulos compensados y esféricos.

7) El error de cierre no debe ser mayor que la tolerancia exigida para el cierre angular.

8) Calculo de cotas de la poligonal y cierre en altura. El error de cierre no debe ser mayor que la tolerancia exigida para el cierre en altura.

$$\Delta h = Di \cdot sen \frac{1}{2} \cdot (Z_2 - Z_1)$$

9) Calculo del azimut inverso, para determinar el azimut de orientación de la poligonal.

$$S \cdot sen\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = \Delta\lambda'' \cdot N_m \cdot \cos\varphi_m \cdot arc1'' \cdot \left[\frac{1 - \dfrac{(\Delta\lambda'' \cdot arc1'')^2}{24} + \dfrac{(\Delta\lambda'' \cdot arc1'')^6}{1920}}{1 - \dfrac{S^2}{24 \cdot N_m{}^2} + \dfrac{S^2}{1920 \cdot N_m{}^4}} \right]$$

$$S \cdot \cos\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = \Delta\varphi'' \cdot \rho_m \cdot \cos\frac{1}{2}\Delta\lambda \cdot arc1'' \cdot \left[\frac{1 - \frac{\rho_m{}^2 \cdot (\Delta\varphi'' \cdot arc1'')^2}{24 \cdot N_m{}^2} + \frac{\rho_m \cdot (\Delta\varphi'' \cdot arc1'')^4}{1920 \cdot N_m{}^4}}{1 - \frac{S^2}{24 \cdot N_m^2} + \frac{S^4}{1920 \cdot N_m^4}}\right]$$

N_m y ρ_m; son radios de curvatura referidos a la latitud media de la línea.

$$tg\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = \frac{\Delta\lambda'' \cdot N_m \cdot \cos\varphi_m \cdot \left[1 - \frac{(\Delta\lambda'' \cdot arc1'')^2}{24} + \frac{(\Delta\lambda'' \cdot arc1'')^6}{1920}\right]}{\Delta\varphi'' \cdot \rho_m \cdot \cos\frac{1}{2}\Delta\lambda \cdot \left[1 - \frac{\rho_m{}^2 \cdot (\Delta\varphi'' \cdot arc1'')^2}{24 \cdot N_m{}^2} + \frac{\rho_m \cdot (\Delta\varphi'' \cdot arc1'')^4}{1920 \cdot N_m{}^4}\right]}$$

$$S_1 = \frac{\Delta\lambda'' \cdot N_m \cdot \cos\varphi_m \cdot arc1'' \cdot \left[1 - \frac{(\Delta\lambda'' \cdot arc1'')^2}{24}\right]}{sen\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right)}$$

$$S_2 = \frac{\Delta\varphi'' \cdot \rho_m \cdot \cos\frac{1}{2}\Delta\lambda \cdot arc1'' \cdot \left[1 - \frac{\rho_m{}^2 \cdot (\Delta\varphi'' \cdot arc1'')^2}{24 \cdot N_m{}^2}\right]}{\cos\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right)}$$

$$S_1 = S_2 = S$$

10) Calculo de las distancias geodésicas de la poligonal.

$$S = \underbrace{Di - \frac{\Delta h^2}{2 \cdot Di} - \frac{\Delta h \cdot hm}{R\alpha}}_{Dnmm} + \frac{Dnmm^3}{24 \cdot R\alpha^2}$$

$$R\alpha = \frac{N_m \cdot \rho_m}{\rho_m \cdot sen^2\alpha + N_m \cdot \cos^2\alpha}$$

11) Con la distancia geodésica y el azimut inicial se procede a calcular las posiciones geográficas preliminares mediante el problema directo.

$$-\Delta\varphi''=\frac{S_{AB}\cdot\cos\alpha_{AB}}{\rho_m\cdot arc1''}+\frac{S_{AB}{}^2\cdot sen^2\alpha_{AB}\cdot tg\varphi_m}{2\cdot N_1\cdot\rho_m\cdot arc1''}-\frac{S_{AB}{}^3\cdot sen^2\alpha_{AB}\cdot\cos\alpha_{AB}\cdot\left(1+3\cdot tg^2\varphi_m\right)}{6\cdot N_1{}^2\cdot\rho_m\cdot arc1''};$$

$$\varphi_B=\varphi_1\pm\Delta\varphi$$

$$-\Delta\lambda''=\frac{S_{AB}\cdot sen\alpha_{AB}}{N_2\cdot\cos\varphi_B\cdot arc1''};\ \lambda_B=\lambda_A\pm\Delta\lambda$$

$$-\Delta\alpha''=\frac{\Delta\lambda''\cdot sen\varphi_m}{\cos\left(\frac{\Delta\varphi}{2}\right)}+\frac{1}{12}\cdot\Delta\lambda''^3\cdot sen\varphi_m\cdot\left(\cos\varphi_m\cdot arc1''\right)^2$$

Calculadas Las coordenadas geográficas preliminares de todos los vértices se procede a compensar verificando que el error de cierre en posición no se encuentre fuera de la tolerancia exigida para el trabajo.

B. CALCULO DEFINITIVO

Del calculo preliminar se obtiene los siguientes antecedentes:

- Cota de todos los vértices.
- Coordenadas de todos los vértices.
- Azimut de todas las líneas.
- Calculo de latitud media de la poligonal.

1) Calculo del exceso esférico.

Con las coordenadas geográficas ya determinadas se obtiene la latitud media de la poligonal, promediando las latitudes de los puntos extremos.

Con φ_m, se calcula ρ y N, con lo cual podremos obtener el exceso esférico del polígono. $E''=\frac{Superficie}{N\cdot\rho\cdot arc1''}$.

2) Reducción de los ángulos horizontales al elipsoide.

(corrección por efecto de la altura de la estación observada)

Esta corrección es aplicable a mediciones efectuadas en planos sobre los 2000 metros.

$$X'' = \frac{H_2 \cdot e^2 \cdot \cos^2 \varphi_2 \cdot \cos\alpha \cdot sen\alpha}{N_m \cdot \left(1 - e^2\right) \cdot arc1''}$$

3) Efectuada la corrección a los ángulos observados por efecto de altura se procederá a cerrar el polígono considerando la esfericidad calculada previamente.

Si el error se encuentra dentro de la tolerancia exigida, se procederá a compensar este error en partes iguales de acuerdo al numero de ángulos que componen la poligonal.

4) Calculo de alturas con distancias geodésicas.

Como se tienen los azimut de las líneas y las coordenadas geográficas preliminares delos vértices se procederá a recalcular las cotas de los vértices de la poligonal.

$$\Delta h = S \cdot tg \frac{1}{2} \cdot \left(Z_2 - Z_1\right) \cdot \left(1 + \frac{h_1}{R\alpha}\right) \cdot \left(1 + \frac{S^2}{12 \cdot R\alpha^2}\right) \cdot \left(1 + \frac{\left|\Delta h'\right|}{2 \cdot R\alpha}\right)$$

5) Cierre polígonos de altura.

Realizado el calculo de cotas de los vértices de la poligonal se deberá compensar este por medio del sistema de proporcionalidad de la distancia recorrida. Esta compensación se realiza siempre que el error de cierre se encuentre dentro de la tolerancia exigida.

Error de cierre < Tolerancia

Factor de compensación (FC) = $\frac{\text{Error de cierre}}{\Sigma\text{Distancia recorrida en kilómetros}}$.

Corrección$_1$ = $FC \cdot S_1$

Corrección$_2$ = $FC \cdot \left(S_1 + S_2\right)$

Corrección$_3$= $FC \cdot \left(S_1 + S_2 + S_3\right)$

Corrección$_4$ = $FC \cdot \left(S_1 + S_2 + S_3 + \ldots\ldots.. + S_n\right)$

6) Calculo de distancias geodésicas definitivas.

Realizada la compensación dela poligonal de altura, se procederá a recalcular las distancias geodésicas.

7) Calculo de posiciones.

Los antecedentes con los cuales contamos son los siguientes:

- Se tiene ya del cálculo preliminar el azimut de orientación.
- Las distancias geodésicas finales.
- Los ángulos horizontales se encuentran reducidos al elipsoide (si correspondiera) y compensados por esfericidad.

Con estos antecedentes se procede a calcular las coordenadas geográficas, mediante el método directo.

8) Compensación y determinación de posiciones finales.

Realizado el calculo de posiciones es necesario verificar el error de cierre, tanto en latitud como en longitud. Si el error en posición es menor que la tolerancia exigida se procederá a compensar y obtener las coordenadas geográficas definitivas de cada uno de los vértices de la poligonal.

Error de cierre < Tolerancia exigida

Error en posición: $\dfrac{\sqrt{\varepsilon\varphi^2 + \varepsilon\lambda^2}}{\Sigma \text{distancia en metros}}$

$\varepsilon\varphi$ y $\varepsilon\lambda$, errores de cierre en latitud y longitud.

$$FC\varphi = \frac{\varepsilon\varphi}{\Sigma S}$$

Correción1= $FC\varphi \cdot S_1$

Corrección2 = $FC\varphi \cdot (S_1 + S_2)$

Corrección3 = $FC\varphi \cdot (S_1 + S_2 + S_3 + ... + S_n)$

$$FC\lambda = \frac{\varepsilon\lambda}{\Sigma S}$$

Correción1= $FC\lambda \cdot S_1$

Corrección2 = $FC\lambda \cdot (S_1 + S_2)$

Corrección 3 = $FC\lambda \cdot (S_1 + S_2 + S_3 + ... + S_n)$

Tabla 5.- Especificaciones técnicas de poligonales electrónicas

	PRIMER ORDEN	SEGUNDO ORDEN 1ª CLASE	SEGUNDO ORDEN 2ª CLASE	TERCER ORDEN 1ª CLASE	TERCER ORDEN 1ª CLASE
DISTANCIA RECOMENDADA ENTRE ESTACIONES	10-20 Km.	4-10 Km.	2-4 Km.	1-2 Km.	0-1 Km.
EXACTITUD DEL TEODOLITO	0,2’’	0,2’’	1’’	1’’	1’’
NUMERO DE REITERACIONES	16 D y T	12 D y T	8 D y T	4 D y T	3 D y T
DIFERENCIA DE LAS MEDICIONES ANGULARES CON RESPECTO A SU PROMEDIO (HORIZONTAL)	4’’	5’’	5’’	5’’	5’’
PRECISIÓN DE LAS OBSERVACIONES CON INSTRUMENTOS DE MEDICION DE DISTANCIAS	1:600.000	1:300.000	1:120.000	1:60.000	1:30.000
OBSERVACIÓN DE ÁNGULOS CENITALES	3 D y T	3 D y T	3 D y T	3 D y T	3 D y T
CIERRE EN POSICIÓN	1:100.000	1:50.000	1:20.000	1:10.000	1:5.000
NUMERO DE ESTACIONES ENTRE VÉRTICES CONOCIDOS	6	8	8-10	10-15	15-20
DIFERENCIAS EN LAS MEDICIONES DE ANGULOS CENITALES CON RESPECTO A SU PROMEDIO	10’’	10’’	10’’	10’’	15’’
CANTIDAD DE ESTACIONES ENTRE PUNTOS ASTRONOMICOS PARA EL CONTROL LAPLACE	6	10-12	15-20	20-25	30-40
CORRECCION DE AZIMUT POR ESTACION NO DEBE EXCEDER	1’’ POR ESTACION	1,5’’	2,0’’	3,0’’	6,0’’

Fuente: IPGH

EJERCICIO DE APLICACIÓN "Calculo de una poligonal de tres lados"

Calcular las coordenadas geográficas de C y su cota.

Elipsoide de referencia : Internacional 1924, $a = 6.378.388m\,;\; f = \frac{1}{297}$

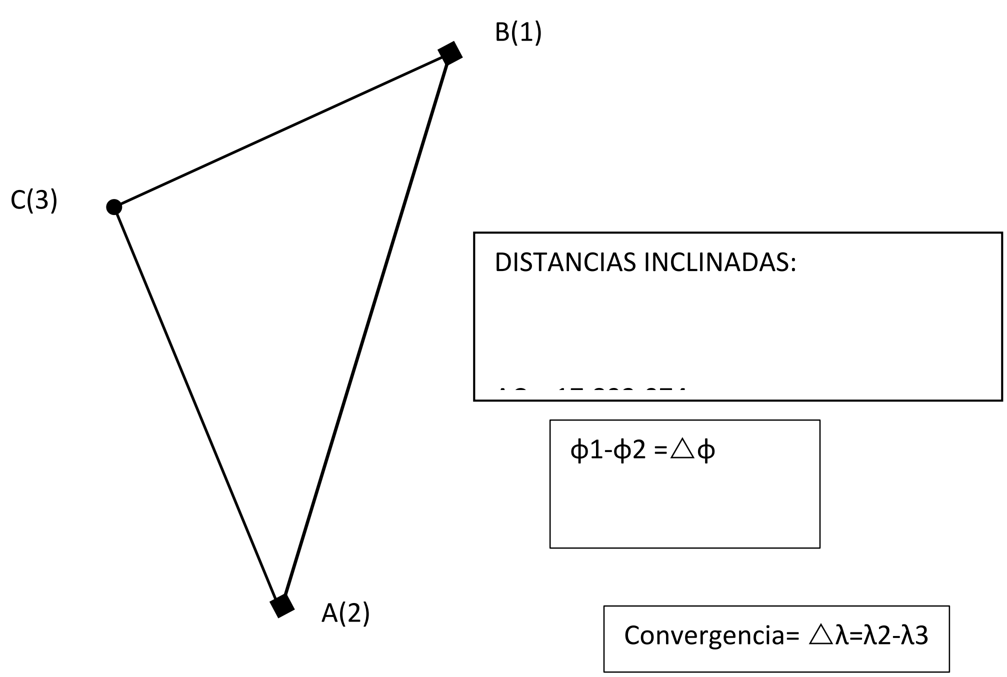
Estación	Punto visado	Angulo horizontal	Angulo vertical	Altura de la señal	Distancia inclinada
A hi = 1,40m	C B	00º 00' 00'' 37º 34' 53,10''	90º 41' 30,20'' ------------	1,25m ------------	17893,07m ------------
B hi = 1,45m	A C	00º 00' 00'' 47º 48' 24,00''	------------ 89º 53' 37,00''	---------- 1,34m	----------- -----------
C hi = 1,46m	B A	00º 00' 00'' 94º 36' 42,00''	90º 13' 51,00'' 90º 41' 30,20''	1,34m 0,00m	14728,70m ------------

$\varphi_A = -31^\circ\,30'\,29{,}65''$ $\qquad \varphi_B = -31^\circ\,18'\,8{,}32''$

$\lambda_A = -70^\circ\,37'\,12{,}37''$ $\qquad \lambda_B = -70^\circ\,32'\,25{,}55''$

$H_A = 3958{,}10m$ $\qquad H_B = 3720{,}89m$

Solución:



Cierre angular de la poligonal

$\angle A: 37^o\,34'\,53{,}10''$

$\angle B: 47^o\,48'\,24{,}00''$ $\quad Error = +0^o\,0'\,0{,}9'' \Rightarrow Error\ unitario = +\dfrac{0{,}9''}{3} = +0{,}3''$

$\angle C: \underline{94^o\,36'\,42{,}00''}$

$\Sigma = 179^o\,59'\,59{,}10''$

$$\text{I orden}\begin{cases}1''\cdot \text{estación}\\ 2''\cdot\sqrt{n} \rightarrow 2''\cdot\sqrt{3} = 3{,}46''; \text{con "n" numero de estaciones.}\end{cases}$$

El error de cierre angular se encuentra dentro la tolerancia por lo tanto se puede compensar.

$\angle A: 37^o\,34'\,53{,}10''$	$\rightarrow +0{,}3''$	$\angle A: 37^o\,34'\,53{,}40''$
$\angle B: 47^o\,48'\,24{,}00''$	$\rightarrow +0{,}3''$	$\angle B: 47^o\,48'\,24{,}30''$
$\angle C: \underline{94^o\,36'\,42{,}00''}$	$\rightarrow +0{,}3''$	$\angle C: \underline{94^o\,36'\,42{,}30''}$
$\Sigma = 179^o\,59'\,59{,}10''$		$\Sigma = 180^o$

Calculo de la superficie de la poligonal

Para el cálculo de la superficie de la poligonal en este caso un triángulo, utilizaremos la siguiente expresión: $Area = \dfrac{\overline{AC}\cdot\overline{BC}}{2}\cdot sen\,\angle C$; siendo $\overline{AC}$ y $\overline{BC}$, distancias horizontales, las cuales serán calculadas con la siguiente expresión: $dhz = \sqrt{Di^2 - \Delta h^2}$

Calculo de Cotas de la Poligonal

Calculo de Δh

<u>A</u>

$Z_1 = 90^o\,41'\,30{,}20''$

$hi_1 = 1{,}40m$

$hb_2 = 1{,}25m$

$H_A = 3958{,}10m$

$Di = 17893{,}074m$

<u>C</u>

$Z_2 = 89^o\,27'\,16{,}00''$

$hi_2 = 1{,}46m$

$hb_1 = 0{,}00m$

Corrección de los ángulos cenitales:

$$\theta_1 = \frac{(hb_2 - hi_1)\cdot senZ_1}{Di \cdot arc1''} = \frac{(1{,}25 - 1{,}40)\cdot sen\,90º\,41'\,30{,}20''}{17893{,}074 \cdot 4{,}848136811 \cdot 10^{-6}} = -1{,}73''$$

$$\theta_2 = \frac{(hb_1 - hi_2)\cdot senZ_2}{Di \cdot arc1''} = \frac{(0{,}00 - 1{,}46)\cdot sen\,89º\,27'\,16{,}00''}{17893{,}074 \cdot 4{,}848136811 \cdot 10^{-6}} = -16{,}83''$$

$$Z_1c = Z_1 \pm \theta_1 = 90º\,41'\,30{,}20'' - 1{,}73'' \Rightarrow Z_1c = 90º\,41'\,28{,}47''$$

$$Z_2c = Z_2 \pm \theta_2 = 89º\,27'\,16{,}00'' - 16{,}83'' \Rightarrow Z_2c = 89º\,26'\,59{,}17''$$

$$\Delta h = Di \cdot sen\frac{1}{2}\cdot(Z_2c - Z_1c) = 17893{,}074 \cdot sen\frac{1}{2}\cdot(89º\,26'\,59{,}17'' - 90º\,41'\,28{,}47'') = -193{,}85m$$

Por lo tanto $\Delta h = -193{,}85m \Rightarrow H_C = H_A \pm \Delta h \;\therefore H_C = 3764{,}25m$

C	B
$Z_1 = 90º\,13'\,51{,}00''$	$Z_2 = 89º\,53'\,37{,}00''$
$hi_1 = 1{,}46m$	$hi_2 = 1{,}45m$
$hb_2 = 1{,}34m$	$hb_1 = 1{,}34m$
$H_C = 3764{,}25m$	
$Di = 14728{,}20m$	

Corrección de los ángulos cenitales:

$$\theta_1 = \frac{(hb_2 - hi_1)\cdot senZ_1}{Di \cdot arc1''} = \frac{(1{,}34 - 1{,}46)\cdot sen\,90º\,13'\,51{,}00''}{14728{,}20 \cdot 4{,}848136811 \cdot 10^{-6}} = -1{,}68''$$

$$\theta_2 = \frac{(hb_1 - hi_2)\cdot senZ_2}{Di \cdot arc1''} = \frac{(1{,}34 - 1{,}45)\cdot sen\,89º\,53'\,37{,}00''}{14728{,}20 \cdot 4{,}848136811 \cdot 10^{-6}} = -1{,}54''$$

$$Z_1c = Z_1 \pm \theta_1 = 90º\,13'\,51{,}00'' - 1{,}68'' \Rightarrow Z_1c = 90º\,13'\,49{,}32''$$

$$Z_2c = Z_2 \pm \theta_2 = 89º\,53'\,37{,}00'' - 1{,}54'' \Rightarrow Z_2c = 89º\,53'\,35{,}46''$$

$$\Delta h = Di \cdot sen\frac{1}{2}\cdot(Z_2c - Z_1c) = 14728{,}70 \cdot sen\frac{1}{2}\cdot(89º\,53'\,35{,}46'' - 90º\,13'\,49{,}32'') = -43{,}34m$$

Por lo tanto $\Delta h = -43{,}34m \Rightarrow H_B = H_C \pm \Delta h \therefore H_B = 3720{,}91m$

H_B fijo = 3720,89*m*
H_B calculado = 3720,91*m*
Error de cierre = -0,02*m*

Tolerancia para el error de cierre de una nivelación trigonométrica.

$$\mathrm{L} = \Sigma_{\text{DISTANCIA INCLINADA}} = 32621{,}774m = 32{,}621774Km.$$

$$\text{I orden} \begin{cases} 0{,}17 \cdot \sqrt{\mathrm{L}} \rightarrow 0{,}1 \cdot \sqrt{32{,}621774} = +/- 0{,}971 \, metros \\ 0{,}15 \cdot \sqrt{\mathrm{L}} \rightarrow 0{,}15 \cdot \sqrt{32{,}621774} = 0{,}8567321139 \approx 0{,}86m \end{cases}$$

El error de cierre de altura se encuentra dentro la tolerancia por lo tanto se puede compensar.

El factor de compensación será: $FC = \dfrac{\text{error de cierre}}{\mathrm{L}}$

$$FC = \frac{-0{,}02}{32621{,}774} = -6{,}130874428 \cdot 10^{-7}$$

Compensación de las cotas:

$$C_n = FC \cdot \Sigma_{DI} \text{ recorrida}$$

$$C_C = FC \cdot 17893{,}074 = -0{,}01097001898 \approx -0{,}01m$$

$$C_B = FC \cdot (17893{,}074 + 14728{,}70) = -0{,}02m$$

Por lo tanto la cota corregida seria:

$$H_{n\,corregida} = H_n \pm C_n$$

$$H_{C\,C} = 3764{,}25 - 0{,}01 \Rightarrow H_{C\,C} = 3764{,}24m$$

$$H_{C\,B} = 3720{,}91 - 0{,}02 \Rightarrow H_{C\,B} = 3720{,}89m$$

Por lo tanto los Δh son:

$$\Delta h_{AC} = H_{C\,C} - H_A \Rightarrow \Delta h_{AC} = 193{,}86m$$

$$\Delta h_{C\,B} = H_{C\,B} - H_{C\,C} \Rightarrow \Delta h_{C\,B} = 43{,}35m$$

Calculo de distancia horizontal:

$$\Delta hz_{AC} = \sqrt{17893{,}074^2 - 193{,}86^2} = 17892{,}024m$$

$$\Delta hz_{CB} = \sqrt{14728{,}70^2 - 43{,}35^2} = 14728{,}636m$$

Calculo de la superficie:

$$A = \frac{\Delta hz_{AC} \cdot \Delta hz_{CB}}{2} \cdot sen\, 94^{o}\, 36'\, 42{,}30'' \Rightarrow A = 131335952{,}50m^2$$

Calculo del exceso esférico:

$$\varphi_m = \frac{\varphi_A + \varphi_B}{2} = -31^o\,24'18{,}985''$$

$$\rho_m = \frac{a\cdot(1-e^2)}{(1-e^2\cdot sen^2\varphi_m)^{3/2}} = \frac{6378388\cdot(1-0{,}0067226700\,22)}{(1-0.0067226700\,22\cdot(sen-31^o\,24'18{,}985'')^2)^{3/2}} = 6352895{,}373m$$

$$N_m = \frac{a}{(1-e^2\cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1-0.0067226700\,22\cdot(sen-31^o\,24'18{,}985'')^2)^{1/2}} = 6384217{,}62m$$

$$E'' = \frac{Area}{N_m\cdot\rho_m\cdot arc1''} = \frac{13133595{,}250}{6384217{,}62\cdot 6352895{,}373\cdot 4{,}848136811\cdot 10^{-6}} = 0{,}67''$$

$$E_U'' = \frac{0{,}67''}{3} = +0{,}22\bar{3}''$$

Cierre angular de la poligonal quedando los ángulos compensados y esféricos

$$\begin{array}{lll} \angle A: 37^o\,34'\,53{,}40'' \rightarrow +0{,}22\bar{3} & \angle A: 37^o\,34'\,53{,}62'' \rightarrow +0{,}01'' & \angle A: 37^o\,34'\,53{,}63'' \\ \angle B: 47^o\,48'\,24{,}30'' \rightarrow +0{,}22\bar{3} & \angle B: 47^o\,48'\,24{,}52'' & \angle B: 47^o\,48'\,24{,}52'' \\ \angle C: \underline{94^o\,36'\,42{,}30''} \rightarrow +0{,}22\bar{3} & \angle C: \underline{94^o\,36'\,42{,}52''} & \angle C: \underline{94^o\,36'\,42{,}52''} \\ \Sigma = 180^o & \Sigma = 180^o\,0'\,0{,}66'' & \Sigma = 180^o\,0'\,0{,}67'' \end{array}$$

Calculo de azimut inverso α_{AB}

$$\begin{array}{ll} \varphi_B: \; -31^o\,18'\,8{,}32'' & \lambda_B: \; -70^o\,32'\,25{,}55'' \\ \varphi_A: - \underline{-31^o\,30'\,29{,}65'} & \lambda_A: - \underline{-70^o\,37'\,12{,}37'} \\ \Delta\varphi = \; 0^o\,12'\,21{,}33'' & \Delta\lambda = \; 0^o\,4'\,46{,}82'' \end{array}$$

$$\Delta\varphi'' = 741{,}33'' \qquad \Delta\lambda'' = 286{,}82''$$

$$\frac{\Delta\varphi''}{2} = 370{,}665'' \qquad \frac{\Delta\lambda''}{2} = 143{,}41''$$

$$\varphi_m = \frac{\varphi_A + \varphi_B}{2} = \frac{-31^o\,18'\,8{,}32'' - 31^o\,30'\,29{,}65''}{2} \Rightarrow \varphi_m = -31^o\,24'18{,}985''$$

$$arc1'' = 4{,}848136811\cdot 10^{-6}$$

$$e^2 = 2\cdot f - f^2 = 2\cdot\frac{1}{297} - \left(\frac{1}{297}\right)^2 \Rightarrow e^2 = 0{,}0067226700\,22$$

$$\rho_m = \frac{a\cdot(1-e^2)}{(1-e^2\cdot sen^2\varphi_m)^{3/2}} = \frac{6378388\cdot(1-0{,}006722670022)}{(1-0.006722670022\cdot(sen-31^\circ\,24'18{,}985'')^2)^{3/2}} = 6352895{,}373m$$

$$N_m = \frac{a}{(1-e^2\cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1-0.006722670022\cdot(sen-31^\circ\,24'18{,}985'')^2)^{1/2}} = 6384217{,}62m$$

$$S_1\cdot sen\left(\alpha+\frac{1}{2}\cdot\Delta\alpha\right) = \Delta\lambda''\cdot N_m\cdot\cos\varphi_m\cdot arc1''\cdot\left[1-\frac{(\Delta\lambda''\cdot arc1'')^2}{24}\right]$$

$$= 286{,}82''\cdot 6384217{,}62\cdot\cos(-31^\circ\,24'18{,}985'')\cdot arc1''\cdot\left[1-\frac{(286{,}82''\cdot arc1'')^2}{24}\right]$$

$$S_1\cdot sen\left(\alpha+\frac{1}{2}\cdot\Delta\alpha\right) = 7576{,}99352$$

$$S_2\cdot\cos\left(\alpha+\frac{1}{2}\cdot\Delta\alpha\right) = \Delta\varphi''\cdot\rho_m\cdot\cos\frac{1}{2}\Delta\lambda\cdot arc1''\cdot\left[1-\frac{\rho_m{}^2\cdot(\Delta\varphi''\cdot arc1'')^2}{24\cdot N_m{}^2}\right]$$

$$= 741{,}33''\cdot 6352895{,}373\cdot\cos(0^\circ\,2'\,23{,}41'')\cdot arc1''\cdot\left[1-\frac{6352895{,}373^2\cdot(741{,}33''\cdot arc1'')^2}{24\cdot 6384217{,}62^2}\right]$$

$$S_2\cdot\cos\left(\alpha+\frac{1}{2}\cdot\Delta\alpha\right) = 22832{,}7283$$

Si $S_1 = S_2$; se tiene :

$$\frac{S_1\cdot sen\left(\alpha+\frac{1}{2}\cdot\Delta\alpha\right)}{S_2\cdot\cos\left(\alpha+\frac{1}{2}\cdot\Delta\alpha\right)} = tg\left(\alpha+\frac{1}{2}\cdot\Delta\alpha\right) = \frac{\Delta\lambda''\cdot N_m\cdot\cos\varphi_m\cdot\left[1-\frac{(\Delta\lambda''\cdot arc1'')^2}{24}\right]}{\Delta\varphi''\cdot\rho_m\cdot\cos\frac{1}{2}\Delta\lambda\cdot\left[1-\frac{\rho_m{}^2\cdot(\Delta\varphi''\cdot arc1'')^2}{24\cdot N_m{}^2}\right]}$$

$$tg\left(\alpha+\frac{1}{2}\cdot\Delta\alpha\right) = \frac{7576{,}99352}{22832{,}7283} = 0{,}3318479255$$

$$\left(\alpha+\frac{1}{2}\cdot\Delta\alpha\right) = tg^{-1}0{,}3318479255 = 18^\circ\,21'\,29{,}94''$$

$$-\Delta\alpha'' = \frac{\Delta\lambda''\cdot sen\varphi_m}{\cos\frac{1}{2}\cdot\Delta\varphi} = \frac{286{,}82''\cdot sen(-31^o\,24'\,18{,}985'')}{\cos(0^o\,6'\,10{,}67'')} \Rightarrow -\Delta\alpha'' = -149{,}4594806''$$

$$\Delta\alpha'' = 149{,}459'' \Rightarrow \frac{\Delta\alpha''}{2} = 74{,}7295'' \Rightarrow \frac{\Delta\alpha}{2} = 0^o\,1'\,14{,}73''$$

$$\alpha = 18^o\,21'\,29{,}94'' + 0^o\,1'\,14{,}73'' \Rightarrow \alpha = 18^o\,22'\,44{,}67''$$

Por lo tanto $\alpha_{AB} = \alpha + 180^o \Rightarrow \alpha_{AB} = 198^o\,22'\,44{,}67''$

$$S_1 = \frac{|7576{,}99352|}{sen\left(\alpha + \frac{1}{2}\cdot\Delta\alpha\right)} = \frac{|7576{,}99352|}{sen(18^o\,21'\,29{,}94'')} \Rightarrow S_1 = 24057{,}11m$$

$$S_2 = \frac{|22832{,}7283|}{\cos\left(\alpha + \frac{1}{2}\cdot\Delta\alpha\right)} = \frac{|22832{,}7283|}{\cos(18^o\,21'\,29{,}94'')} \Rightarrow S_2 = 24057{,}11m$$

$S_1 = S_2$; Por lo tanto la distancia geodésica es: $S_{AB} = 24057{,}11m$

Calculo de las distancias geodésicas

Calculo de distancia geodésica S_{AC}

$$\varphi_m = \varphi_A = -31^o\,30'\,29{,}65''$$

$$\rho_m = \frac{a\cdot(1-e^2)}{(1-e^2\cdot sen^2\varphi_m)^{3/2}} = \frac{6378388\cdot(1-0{,}0067226700\,22)}{(1-0.0067226700\,22\cdot(sen-31^o\,30'\,29{,}65'')^2)^{3/2}} = 6352998{,}058m$$

$$N_m = \frac{a}{(1-e^2\cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1-0.0067226700\,22\cdot(sen-31^o\,30'\,29{,}65'')^2)^{1/2}} = 6384252{,}017m$$

$$\alpha_{AC} = \alpha_{AB} - \angle A = 198^o\,22'\,44{,}67'' - 37^o\,34'\,53{,}63'' \Rightarrow \alpha_{AC} = 160^o\,47'\,51{,}04''$$

$$R\alpha = \frac{N_m \cdot \rho_m}{\rho_m \cdot sen^2\alpha_{AC} + N_m \cdot \cos^2 \alpha_{AC}}$$

$$= \frac{6384252{,}017 \cdot 6352998{,}058}{6352998{,}058 \cdot (sen160^\circ\, 47'\, 51{,}04'')^2 + 6384252{,}017 \cdot (\cos 160^\circ\, 47'\, 51{,}04'')^2}$$

$$R\alpha = 6356364{,}35m$$

$$Dh = \sqrt{Di^2 - \Delta h^2} = \sqrt{17893{,}074^2 - 193{,}86^2} = 17892{,}024m$$

$$hm = \frac{H_A + H_C}{2} = \frac{3958{,}10 + 3764{,}24}{2} = 3861{,}17m$$

$$Cnmm = \frac{-Dh \cdot hm}{R\alpha} = -\frac{17892{,}024 \cdot 3861{,}17}{6356364{,}35} = -10{,}869m$$

$$Dnmm = 17892{,}024 - 10{,}869 = 17881{,}155m$$

$$Cc = \frac{Dnmm^3}{24 \cdot R\alpha^2} = \frac{117881{,}155^3}{24 \cdot 6356364{,}35^2} = 0{,}006m$$

$$S = 17881{,}155 + 0{,}006 \Rightarrow S = 17881{,}161m$$

Calculo de distancia geodésica S_{CB}

$$\varphi_m = \varphi_A = -31^\circ\, 30'\, 29{,}65''$$

$$\rho_m = \frac{a \cdot (1 - e^2)}{(1 - e^2 \cdot sen^2\varphi_m)^{3/2}} = \frac{6378388 \cdot (1 - 0{,}0067226700{,}22)}{(1 - 0.0067226700{,}22 \cdot (sen - 31^\circ\, 30'\, 29{,}65'')^2)^{3/2}} = 6352998{,}058m$$

$$N_m = \frac{a}{(1 - e^2 \cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1 - 0.0067226700{,}22 \cdot (sen - 31^\circ\, 30'\, 29{,}65'')^2)^{1/2}} = 6384252{,}017m$$

$$\alpha_{CB} = \alpha_{AC} + 180^\circ - \angle C = 160^\circ\, 47'\, 51{,}04'' + 180^\circ - 94^\circ\, 36'\, 42{,}52'' \Rightarrow \alpha_{CB} = 246^\circ\, 11'\, 8{,}52''$$

$$R\alpha = \frac{N_m \cdot \rho_m}{\rho_m \cdot sen^2\alpha_{CB} + N_m \cdot \cos^2 \alpha_{CB}}$$

$$= \frac{6384252{,}017 \cdot 6352998{,}058}{6352998{,}058 \cdot \left(sen 246^\circ 11' 8{,}52''\right)^2 + 6384252{,}017 \cdot \left(\cos 246^\circ 11' 8{,}52''\right)^2}$$

$$R\alpha = 6379135{,}622 m$$

$$Dh = \sqrt{Di^2 - \Delta h^2} = \sqrt{14728{,}70^2 - 43{,}35^2} = 14728{,}636 m$$

$$hm = \frac{H_B + H_C}{2} = \frac{3720{,}89 + 3764{,}24}{2} = 3742{,}565 m$$

$$Cnmm = \frac{-Dh \cdot hm}{R\alpha} = -\frac{14728{,}636 \cdot 3742{,}565}{6379135{,}622} = -8{,}641 m$$

$$Dnmm = 14728{,}636 - 8{,}641 = 14719{,}995 m$$

$$Cc = \frac{Dnmm^3}{24 \cdot R\alpha^2} = \frac{14719{,}995^3}{24 \cdot 6379135{,}622^2} = 0{,}003 m$$

$$S = 14719{,}995 + 0{,}003 \Rightarrow S = 14719{,}998 m$$

Calculo de posición por el problema directo
(A $\rightarrow$ C)

Calculo de latitud

$$-\Delta\varphi'' = \underbrace{\frac{S \cdot \cos\alpha_{AC}}{\rho_m \cdot arc1''}}_{I} + \underbrace{\frac{S^2 \cdot sen^2\alpha_{AC} \cdot tg\varphi_m}{2 \cdot N_1 \cdot \rho_m \cdot arc1''}}_{II} - \underbrace{\frac{S^3 \cdot sen^2\alpha_{AC} \cdot \cos\alpha_{AC} \cdot \left(1 + 3 \cdot tg^2\varphi_m\right)}{6 \cdot N_1^2 \cdot \rho_m \cdot arc1''}}_{III}$$

Iteración 1

$$\alpha_{AC} = 160^\circ 47' 51{,}05''$$
$$S = 17881{,}161 m$$
$$\varphi_{m1} = \varphi_A = -31^\circ 30' 29{,}65''$$

$$\rho_{m1} = \frac{a \cdot (1 - e^2)}{(1 - e^2 \cdot sen^2 \varphi_{m1})^{3/2}} = \frac{6378388 \cdot (1 - 0,006722670022)}{(1 - 0.006722670022 \cdot (sen -31^\circ 30' 29,65'')^2)^{3/2}} = 6352998,058m$$

$$N_1 = \frac{a}{(1 - e^2 \cdot sen^2 \varphi_{m1})^{1/2}} = \frac{6378388}{(1 - 0.006722670022 \cdot (sen -31^\circ 30' 29,65'')^2)^{1/2}} = 6384252,017m$$

$$I_{(1)} = \frac{S \cdot \cos \alpha_{AC}}{\rho_{m1} \cdot arc1''} = \frac{17881,161 \cdot \cos 160^\circ 47' 51,05''}{6352998,058 \cdot 4,848136811 \cdot 10^{-6}} = -548,252557''$$

$$II_{(1)} = \frac{S^2 \cdot sen^2 \alpha_{AC} \cdot tg \varphi_{m1}}{2 \cdot N_1 \cdot \rho_{m1} \cdot arc1''} = \frac{17881,161^2 \cdot (sen\,160^\circ 47' 51,05'')^2 \cdot tg -31^\circ 30' 29,65''}{2 \cdot 6384252,017 \cdot 6352998,058 \cdot 4,848136811 \cdot 10^{-6}}$$

$$II_{(1)} = -0,05391445956''$$

$$III_{(1)} = \frac{S^3 \cdot sen^2 \alpha_{AC} \cdot \cos \alpha_{AC} \cdot (1 + 3 \cdot tg^2 \varphi_{m1})}{6 \cdot N_1^{\,2} \cdot \rho_{m1} \cdot arc1''}$$

$$= \frac{17881,161^3 \cdot (sen\,160^\circ 47' 51,05'')^2 \cdot \cos 160^\circ 47' 51,05'' \cdot (1 + 3 \cdot (tg -31^\circ 30' 29,65'')^2)}{6 \cdot 6384252,017^2 \cdot 6352998,058 \cdot 4,848136811 \cdot 10^{-6}}$$

$$III_{(1)} = -0,0001649597033''$$

$$-\Delta\varphi'' = I_{(1)} + II_{(1)} - II_{(1)} = -548,252'' - 0,054'' + 0,000 = -548,306''$$

$$\therefore \Delta\varphi = 0^\circ 9' 8,31''$$

$$\varphi_C = \varphi_A + \Delta\varphi \Rightarrow \varphi_C = -31^\circ 30' 29,65'' + 0^\circ 9' 8,31'' \;\therefore\; \varphi_C = -31^\circ 21' 21,34''$$

$$\varphi_{m2} = \frac{\varphi_A + \varphi_C}{2} = \frac{-31^\circ 30' 29,65'' - 31^\circ 21' 21,34''}{2} = -31^\circ 25' 55,50''$$

Iteración 2

$$\rho_{m2} = \frac{a \cdot (1-e^2)}{(1-e^2 \cdot sen^2\varphi_{m2})^{3/2}} = \frac{6378388 \cdot (1-0,0067226700022)}{(1-0.0067226700022 \cdot (sen -31^{o}\,25'\,55{,}50'')^2)^{3/2}} = 6352922{,}092m$$

$$N_1 = 6384252{,}017m$$

$$I_{(2)} = \frac{S \cdot \cos\alpha_{AC}}{\rho_{m2} \cdot arc1''} = \frac{17881{,}161 \cdot \cos 160^{o}\,47'\,51{,}05''}{6352922{,}092 \cdot 4{,}8481368111 \cdot 10^{-6}} = -548{,}25911128'$$

$$II_{(2)} = \frac{S^2 \cdot sen^2\alpha_{AC} \cdot tg\varphi_{m2}}{2 \cdot N_1 \cdot \rho_{m2} \cdot arc1''} = \frac{17881{,}161^2 \cdot (sen\,160^{o}\,47'\,51{,}05'')^2 \cdot tg -31^{o}\,25'\,55{,}50''}{2 \cdot 6384252{,}017 \cdot 6352922{,}092 \cdot 4{,}8481368111 \cdot 10^{-6}}$$

$$II_{(2)} = -0{,}05375440803''$$

$$III_{(2)} = \frac{S^3 \cdot sen^2\alpha_{AC} \cdot \cos\alpha_{AC} \cdot (1+3 \cdot tg^2\varphi_{m2})}{6 \cdot N_1^{\,2} \cdot \rho_{m2} \cdot arc1''}$$

$$= \frac{17881{,}161^3 \cdot (sen\,160^{o}\,47'\,51{,}05'')^2 \cdot \cos 160^{o}\,47'\,51{,}05'' \cdot (1+3 \cdot (tg -31^{o}\,25'\,55{,}50'')^2)}{6 \cdot 6384252{,}017^2 \cdot 6352922{,}092 \cdot 4{,}8481368111 \cdot 10^{-6}}$$

$$III_{(2)} = -0{,}0001644413545''$$

$$-\Delta\varphi'' = I_{(2)} + II_{(2)} - III_{(2)} = -548{,}259'' - 0{,}054'' + 0{,}000 = -548{,}313''$$

$$\therefore \Delta\varphi = 0^{o}\,9'\,8{,}31''$$

$$\varphi_C = \varphi_A + \Delta\varphi \Rightarrow \varphi_C = -31^{o}\,30'\,29{,}65'' + 0^{o}\,9'\,8{,}31'' \;\therefore\; \varphi_C = -31^{o}\,21'\,21{,}34''$$

$$\varphi_{m3} = \frac{\varphi_A + \varphi_C}{2} = \frac{-31^{o}\,30'\,29{,}65'' - 31^{o}\,21'\,21{,}34''}{2} = -31^{o}\,25'\,55{,}50''$$

$\varphi_{m2} = \varphi_{m3}$; Por lo tanto se cumple la convergencia

Calculo de longitud

$$N_2 = \frac{a}{\left(1 - e^2 \cdot sen^2\varphi_C\right)^{1/2}} = \frac{6378388}{\left(1 - 0.00672267002 \cdot (sen - 31^{\circ}\,21'\,21{,}34'')^2\right)^{1/2}} = 6384201{,}158m$$

$$-\Delta\lambda'' = \frac{S \cdot sen\alpha_{AC}}{N_2 \cdot \cos\varphi_C \cdot arc1''} = \frac{17881{,}161 \cdot sen\,160^{\circ}\,47'\,51{,}05''}{6384201{,}158 \cdot \cos - 31^{\circ}\,21'\,21{,}34'' \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$= 222{,}5128601'' \Rightarrow \Delta\lambda''' = -222{,}51'' \therefore \Delta\lambda = -0^{\circ}\,3'\,42{,}51''$$

Por lo tanto la longitud es:

$$\lambda_C = \lambda_A + \Delta\lambda = -70^{\circ}\,37'\,12{,}37'' - 0^{\circ}\,3'\,42{,}51'' \therefore \lambda_C = -70^{\circ}\,40'\,54{,}88''$$

Calculo de azimut inverso α_{CA}

$$\alpha_{CA} = \alpha_{AC} + 180^{\circ} \pm \Delta\alpha$$

$$\Delta\varphi = \varphi_A - \varphi_C = -0^{\circ}\,9'\,8{,}31'' \Rightarrow \frac{\Delta\varphi}{2} = -0^{\circ}\,4'\,34{,}155'$$

$$\Delta\lambda = \lambda_A - \lambda_C = 222{,}51''$$

$$-\Delta\alpha'' = \frac{\Delta\lambda'' \cdot sen\varphi_m}{\cos\frac{1}{2} \cdot \Delta\varphi} = \frac{222{,}51'' \cdot sen(-31^{\circ}\,25'\,55{,}50'')}{\cos(-0^{\circ}\,4'\,34{,}155'')} \Rightarrow -\Delta\alpha'' = -116{,}0362871''$$

$$\Delta\alpha'' = 116{,}04'' \Rightarrow \Delta\alpha = 0^{\circ}\,1'\,56{,}04''$$

$$\alpha_{CA} = \alpha_{AC} + 180^{\circ} \pm \Delta\alpha = 160^{\circ}\,47'\,51{,}05'' + 180^{\circ} + 0^{\circ}\,1'\,56{,}04'' \Rightarrow \alpha_{CA} = 340^{\circ}\,49'\,47{,}09''$$

Calculo de posición por el problema directo (C $\rightarrow$ B)

Calculo de latitud

$$-\Delta\varphi'' = \underbrace{\frac{S \cdot \cos\alpha_{CB}}{\rho_m \cdot arc1''}}_{I} + \underbrace{\frac{S^2 \cdot sen^2\alpha_{CB} \cdot tg\varphi_m}{2 \cdot N_1 \cdot \rho_m \cdot arc1''}}_{II} - \underbrace{\frac{S^3 \cdot sen^2\alpha_{CB} \cdot \cos\alpha_{CB} \cdot \left(1 + 3 \cdot tg^2\varphi_m\right)}{6 \cdot N_1{}^2 \cdot \rho_m \cdot arc1''}}_{III}$$

Iteración 1

$$\alpha_{CB} = \alpha_{CA} - \angle C = 340^o\,49'\,47{,}09'' - 94^o\,36'\,42{,}52'' \Rightarrow \alpha_{CB} = 246^o\,13'\,4{,}57''$$

$$S = 14719{,}998\,m$$

$$\varphi_{m1} = \varphi_C = -31^o\,21'\,21{,}34''$$

$$\rho_{m1} = \frac{a\cdot(1-e^2)}{(1-e^2\cdot sen^2\varphi_{m1})^{3/2}} = \frac{6378388\cdot(1-0{,}0067226700\,22)}{(1-0.0067226700\,22\cdot(sen -31^o\,21'\,21{,}34'')^2)^{3/2}} = 6352846{,}228\,m$$

$$N_1 = \frac{a}{(1-e^2\cdot sen^2\varphi_{m1})^{1/2}} = \frac{6378388}{(1-0.0067226700\,22\cdot(sen -31^o\,21'\,21{,}34'')^2)^{1/2}} = 6384201{,}158\,m$$

$$I_{(1)} = \frac{S\cdot\cos\alpha_{CB}}{\rho_{m1}\cdot arc1''} = \frac{14719{,}998\cdot\cos 246^o\,13'\,4{,}57''}{6352846{,}228\cdot 4{,}848136811\cdot 10^{-6}} = -192{,}7296143'$$

$$II_{(1)} = \frac{S^2\cdot sen^2\alpha_{CB}\cdot tg\varphi_{m1}}{2\cdot N_1\cdot\rho_{m1}\cdot arc1''} = \frac{14719{,}998^2\cdot(sen\,246^o\,13'\,4{,}57'')^2\cdot tg -31^o\,21'\,21{,}34''}{2\cdot 6384201{,}158\cdot 6352846{,}228\cdot 4{,}848136811\cdot 10^{-6}}$$

$$II_{(1)} = -0{,}2811411379'$$

$$III_{(1)} = \frac{S^3\cdot sen^2\alpha_{CB}\cdot\cos\alpha_{CB}\cdot(1+3\cdot tg^2\varphi_{m1})}{6\cdot N_1^2\cdot\rho_{m1}\cdot arc1''}$$

$$= \frac{14719{,}998^3\cdot(sen\,246^o\,13'\,4{,}57'')^2\cdot\cos 246^o\,13'\,4{,}57''\cdot(1+3\cdot(tg -31^o\,21'\,21{,}34'')^2)}{6\cdot 6384201{,}158^2\cdot 6352846{,}228\cdot 4{,}848136811\cdot 10^{-6}}$$

$$III_{(1)} = -0{,}0003022806431''$$

$$-\Delta\varphi'' = I_{(1)} + II_{(1)} - II_{(1)} = -192{,}730'' - 0{,}281'' + 0{,}000 = -193{,}011''$$

$$\therefore \Delta\varphi = 0^o\,3'\,13{,}01''$$

$$\varphi_B = \varphi_C + \Delta\varphi \Rightarrow \varphi_B = -31^o\,21'\,21{,}34'' + 0^o\,3'\,13{,}01'' \;\therefore \varphi_B = -31^o\,18'\,8{,}33''$$

Iteración 2

$$\varphi_{m2} = \frac{\varphi_B + \varphi_C}{2} = \frac{-31^o\,18'\,8{,}33'' - 31^o\,21'\,21{,}34''}{2} = -31^o\,19'\,44{,}835''$$

$$\rho_{m2} = \frac{a \cdot (1-e^2)}{(1-e^2 \cdot sen^2\varphi_{m2})^{3/2}} = \frac{6378388 \cdot (1-0,006722670022)}{(1-0.006722670022 \cdot (sen -31^{\circ}19'44,835'')^2)^{3/2}} = 6352819,549m$$

$$N_1 = 6384201,158m$$

$$I_{(2)} = \frac{S \cdot \cos\alpha_{CB}}{\rho_{m2} \cdot arc1''} = \frac{14719,998 \cdot \cos 246^{\circ}13'4,57''}{6352819,549 \cdot 4,848136811 \cdot 10^{-6}} = -192,7304237''$$

$$II_{(2)} = \frac{S^2 \cdot sen^2\alpha_{CB} \cdot tg\varphi_{m2}}{2 \cdot N_1 \cdot \rho_{m2} \cdot arc1''} = \frac{14719,998^2 \cdot (sen\, 246^{\circ}13'4,57'')^2 \cdot tg -31^{\circ}19'44,835''}{2 \cdot 6384201,158 \cdot 6352819,549 \cdot 4,848136811 \cdot 10^{-6}}$$

$$II_{(2)} = -0,2808463838'$$

$$III_{(2)} = \frac{S^3 \cdot sen^2\alpha_{CB} \cdot \cos\alpha_{CB} \cdot (1+3 \cdot tg^2\varphi_{m2})}{6 \cdot N_1^{\,2} \cdot \rho_{m2} \cdot arc1''}$$

$$= \frac{14719,998^3 \cdot (sen\, 246^{\circ}13'4,57'')^2 \cdot \cos 246^{\circ}13'4,57'' \cdot (1+3 \cdot (tg -31^{\circ}19'44,835'')^2)}{6 \cdot 6384201,158^2 \cdot 6352819,549 \cdot 4,848136811 \cdot 10^{-6}}$$

$$III_{(2)} = -0,0003019467563''$$

$$-\Delta\varphi'' = I_{(2)} + II_{(2)} - III_{(2)} = -192,730'' - 0,281'' + 0,000 = -193,011''$$

$$\therefore \Delta\varphi = 0^{\circ}3'13,01''$$

$$\varphi_B = \varphi_C + \Delta\varphi \Rightarrow \varphi_B = -31^{\circ}21'21,34'' + 0^{\circ}3'13,01'' \;\therefore \varphi_B = -31^{\circ}18'8,33''$$

$$\varphi_{m3} = \frac{\varphi_B + \varphi_C}{2} = \frac{-31^{\circ}18'8,33'' - 31^{\circ}21'21,34''}{2} = -31^{\circ}19'44,835''$$

$\varphi_{m2} = \varphi_{m3}$; Por lo tanto se cumple la convergencia.

Calculo de longitud

$$N_2 = \frac{a}{(1-e^2 \cdot sen^2\varphi_B)^{1/2}} = \frac{6378388}{(1-0.006722670022 \cdot (sen -31^{\circ}18'8,33'')^2)^{1/2}} = 6384183,288m$$

$$-\Delta\lambda'' = \frac{S \cdot sen\,\alpha_{CB}}{N_2 \cdot \cos\varphi_B \cdot arc1''} = \frac{14719{,}998 \cdot sen\,246^{\circ}\,13'\,4{,}57''}{6384183{,}288 \cdot \cos -31^{\circ}\,18'\,8{,}33'' \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$= -509{,}3413639'' \Rightarrow \Delta\lambda''' = 509{,}34'' \therefore \Delta\lambda = 0^{\circ}\,8'\,29{,}34''$$

Por lo tanto la longitud es:

$$\lambda_B = \lambda_C + \Delta\lambda = -70^{\circ}\,40'\,54{,}88'' + 0^{\circ}\,8'\,29{,}34'' \therefore \lambda_B = -70^{\circ}\,32'\,25{,}54''$$

$$\varphi_{B\,FIJO} = -31^{\circ}\,18'\,8{,}32'' \qquad \lambda_{B\,FIJO} = -70^{\circ}\,32'\,25{,}55''$$

$$\varphi_{B\,CALCULADO} = \underline{-31^{\circ}\,18'\,8{,}33''} \qquad \lambda_{B\,CALCULADO} = \underline{-70^{\circ}\,32'\,25{,}54''}$$

$$\Delta\varphi'' = 0{,}01'' \qquad \Delta\lambda'' = -0{,}01''$$

Error en posición = $\dfrac{\sqrt{\varepsilon\varphi^2 + \varepsilon\lambda^2}}{\Sigma s}$

Arco de paralelo = $N_m \cdot \cos\varphi_m \cdot arc\,1''$

Arco de meridiano = $\rho_m \cdot arc\,1''$

$$\varphi_A = -31^{\circ}\,30'\,29{,}65''$$
$$\varphi_B = -31^{\circ}\,18'\,8{,}33''$$
$$\varphi_C = \underline{-31^{\circ}\,21'\,21{,}34''}$$
$$\varphi_m = -31^{\circ}\,23'\,19{,}77$$

$$\rho_m = \frac{a \cdot (1 - e^2)}{(1 - e^2 \cdot sen^2\varphi_m)^{3/2}} = \frac{6378388 \cdot (1 - 0{,}0067226700\,22)}{(1 - 0.0067226700\,22 \cdot (sen -31^{\circ}\,23'\,19{,}77'')^2)^{3/2}} = 6352878{,}987m$$

$$N_m = \frac{a}{(1 - e^2 \cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1 - 0.0067226700\,22 \cdot (sen -31^{\circ}\,23'\,19{,}77'')^2)^{1/2}} = 6384212{,}131m$$

Arco de paralelo = $N_m \cdot \cos\varphi_m \cdot arc\,1'' = 6384212{,}131 \cdot \cos -31^{\circ}\,23'\,19{,}77'' \cdot arc\,1'' = 26{,}422m$

Arco de meridiano = $\rho_m \cdot arc\,1'' = 6352878{,}987 \cdot arc\,1'' = 30{,}800m$

$$\varepsilon\varphi = \Delta\varphi'' \cdot \text{Arco de meridiano} = 0{,}01'' \cdot 30{,}800 = 0{,}308$$
$$\varepsilon\lambda = \Delta\lambda'' \cdot \text{Arco de paralelo} = -0{,}01'' \cdot 26{,}422 = -0{,}264$$

Error lineal = $\sqrt{\varepsilon\varphi^2 + \varepsilon\lambda^2} = \sqrt{(0{,}308)^2 + (-0{,}264)^2} = 0{,}406m$

$\Sigma s = S_{AC} + S_{CB} = 17881{,}161 + 14719{,}998 = 32601{,}159m$

Error de posición = $\frac{\text{Error lineal}}{\Sigma s} = \frac{0{,}406}{32601{,}159} = \frac{0{,}406 / 0{,}406}{32601{,}159 / 0{,}406} = \frac{1}{80298{,}42118}$

Error en posición = $\frac{\sqrt{\varepsilon\varphi^2 + \varepsilon\lambda^2}}{\Sigma s} \leq \frac{1}{40.000}$; para que el error de posición se encuentre dentro de la tolerancia de I orden.

Error en posición = $\frac{1}{80298{,}42118} < \frac{1}{40.000} \Rightarrow 0{,}00001245354498 < 0{,}000025$

El error en posición se encuentra dentro de tolerancia por lo tanto se puede compensar.

Factor de compensación; $FC_{\varphi} = \frac{\Delta\varphi''}{\Sigma S}, FC_{\lambda} = \frac{\Delta\lambda''}{\Sigma S}$

Corrección $n = FC_{\varphi} \cdot (S_1 + S_2 + ... + S_n)$ $\rightarrow$ corrección $n = FC_{\lambda} \cdot (S_1 + S_2 + ... + S_n)$

$$FC_{\varphi} = \frac{0{,}01}{32601{,}159} = 3{,}067375611 \cdot 10^{-7}$$

$$FC_{\lambda} = \frac{-0{,}01}{32601{,}159} = -3{,}067375611 \cdot 10^{-7}$$

Corrección $\varphi_C = -31^{\circ}\ 21'\ 21{,}34'' + FC_{\varphi} \cdot (17881{,}161) = -31^{\circ}\ 21'\ 21{,}34'' + 0{,}0055''$

$\varphi_C = -31^{\circ}\ 21'\ 21{,}33''$

Corrección $\varphi_B = -31^{\circ}\ 18'\ 8{,}33'' + FC_{\varphi} \cdot (17881{,}161 + 14719{,}998) = -31^{\circ}\ 18'\ 8{,}33'' + 0{,}01''$

$\varphi_B = -31^{\circ}\ 18'\ 8{,}32''$

Corrección $\lambda_C = -70^{\circ}\ 40'\ 54{,}88'' - FC_{\lambda} \cdot (17881{,}161) = -70^{\circ}\ 40'\ 54{,}88'' - 0{,}0055''$

$\lambda_C = -70^{\circ}\ 40'\ 54{,}89''$

Corrección $\lambda_B = -70^{\circ}\ 32'\ 25{,}54'' - FC_{\lambda} \cdot (17881{,}161 + 14719{,}998) = -70^{\circ}\ 32'\ 25{,}54'' - 0{,}01''$

$\lambda_B = -70^{\circ}\ 32'\ 25{,}55''$

CALCULO DEFINITIVO

Calculo del exceso esférico

$$\varphi_A = -31^{\circ}\,30'\,29{,}65''$$
$$\varphi_B = -31^{\circ}\,18'\,8{,}32''$$
$$\varphi_C = -31^{\circ}\,21'\,21{,}33''$$
$$\varphi_m = -31^{\circ}\,23'\,19{,}77$$

$$\rho_m = \frac{a\cdot\left(1-e^2\right)}{\left(1-e^2\cdot sen^2\varphi_m\right)^{3/2}} = \frac{6378388\cdot\left(1-0{,}0067226700\,22\right)}{\left(1-0.0067226700\,22\cdot\left(sen-31^{\circ}\,23'19{,}77''\right)^2\right)^{3/2}} = 6352878{,}987m$$

$$N_m = \frac{a}{\left(1-e^2\cdot sen^2\varphi_m\right)^{1/2}} = \frac{6378388}{\left(1-0.0067226700\,22\cdot\left(sen-31^{\circ}\,23'19{,}77''\right)^2\right)^{1/2}} = 6384212{,}131m$$

$$E'' = \frac{Area}{N_m\cdot\rho_m\cdot arc1''} = \frac{13133595{,}250}{6384212{,}131\cdot 6352878{,}987\cdot 4{,}848136811\cdot 10^{-6}} = 0{,}67''$$

$$E_U'' = \frac{0{,}67''}{3} = +0{,}22\bar{3}''$$

REDUCCIÓN DE LOS ÁNGULOS HORIZONTALES AL ELIPSOIDE

Corrección por efecto de la altura de la estación observada.

$$X'' = \frac{H_2\cdot e^2\cdot\cos^2\varphi_2\cdot\cos\alpha\cdot sen\alpha}{N_m\cdot\left(1-e^2\right)\cdot arc1''}$$

$$\varphi_A = -31^{\circ}\,30'\,29{,}65'' \rightarrow \varphi_C = -31^{\circ}\,21'\,21{,}33'';\, H_C = 3764{,}24m$$
$$\alpha_{AC} = 160^{\circ}\,47'\,51{,}05''$$
$$\varphi_m = \frac{\varphi_A+\varphi_C}{2} = -31^{\circ}\,25'\,55{,}49''$$
$$N_m = \frac{a}{\left(1-e^2\cdot sen^2\varphi_m\right)^{1/2}} = \frac{6378388}{\left(1-0.0067226700\,22\cdot\left(sen-31^{\circ}\,25'\,55{,}49''\right)^2\right)^{1/2}} = 6384226{,}57m$$

$$X_C'' = \frac{H_C \cdot e^2 \cdot \cos^2 \varphi_C \cdot \cos \alpha_{AC} \cdot sen\alpha_{AC}}{N_m \cdot (1-e^2) \cdot arc1''}$$

$$X_C'' = \frac{3764{,}24 \cdot 0{,}0067226700{,}22 \cdot \cos^2(-31^\circ 21' 21{,}33'') \cdot \cos 160^\circ 47' 51{,}05'' \cdot sen160^\circ 47' 51{,}05''}{638422{,}57 \cdot (1-0{,}0067226700{,}22) \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$X_C'' = -0{,}186''$$

$\varphi_A = -31^\circ 30' 29{,}65'' \rightarrow \varphi_B = -31^\circ 18' 8{,}32''; H_B = 3720{,}89m$

$\alpha_{AB} = 198^\circ 22' 44{,}67''$

$$\varphi_m = \frac{\varphi_A + \varphi_B}{2} = -31^\circ 24' 18{,}985''$$

$$N_m = \frac{a}{(1-e^2 \cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1-0.0067226700{,}22 \cdot (sen-31^\circ 24' 18{,}985'')^2)^{1/2}} = 6384217{,}62m$$

$$X_B'' = \frac{H_B \cdot e^2 \cdot \cos^2 \varphi_B \cdot \cos \alpha_{AB} \cdot sen\alpha_{AB}}{N_m \cdot (1-e^2) \cdot arc1''}$$

$$X_B'' = \frac{3720{,}89 \cdot 0{,}0067226700{,}22 \cdot \cos^2(-31^\circ 18' 8{,}32'') \cdot \cos 198^\circ 22' 29{,}65'' \cdot sen198^\circ 22' 29{,}65''}{6384217{,}62 \cdot (1-0{,}0067226700{,}22) \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$X_B'' = 0{,}178''$$

Ángulos observados en terreno

Estación	Punto visado	Angulo leído	X’’	Angulo corregido	Angulo reducido
A	C	00º 00’ 00’’	-0,186’’	359º 59’ 59,81’’	00º 00’ 00’’
	B	37º 34’ 53,1’’	0,178’’	37º 34’ 53,28’’	37º 34’ 53,47’’

$\varphi_C = -31^\circ 21' 21{,}33'' \rightarrow \varphi_B = -31^\circ 18' 8{,}32''; H_B = 3720{,}89m$

$\alpha_{CB} = 246^\circ 13' 4{,}57''$

$$\varphi_m = \frac{\varphi_C + \varphi_B}{2} = -31^\circ 19' 44{,}825''$$

$$N_m = \frac{a}{(1-e^2 \cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1-0.0067226700{,}22 \cdot (sen-31^\circ 19' 44{,}825'')^2)^{1/2}} = 6384192{,}22m$$

$$X_B'' = \frac{H_B \cdot e^2 \cdot \cos^2 \varphi_B \cdot \cos\alpha_{CB} \cdot sen\alpha_{CB}}{N_m \cdot (1-e^2) \cdot arc1''}$$

$$X_B'' = \frac{3720{,}89 \cdot 0{,}0067226700\,22 \cdot \cos^2(-31^o\,18'\,8{,}32'') \cdot \cos 246^o\,13'\,4{,}57'' \cdot sen 246^o\,13'\,4{,}57''}{6384192{,}22 \cdot (1-0{,}0067226700\,22) \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$X_B'' = 0{,}219''$$

$$\varphi_C = -31^o\,21'\,21{,}33'' \rightarrow \varphi_A = -31^o\,30'\,29{,}65''\,;H_B = 3958{,}10m$$
$$\alpha_{CA} = 340^o\,49'\,47{,}09''$$
$$\varphi_m = \frac{\varphi_C + \varphi_A}{2} = -31^o\,25'\,55{,}49''$$
$$N_m = \frac{a}{(1-e^2 \cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1-0.0067226700\,22 \cdot (sen-31^o\,25'\,55{,}49'')^2)^{1/2}} = 6384226{,}17m$$

$$X_A'' = \frac{H_A \cdot e^2 \cdot \cos^2 \varphi_A \cdot \cos\alpha_{CA} \cdot sen\alpha_{CA}}{N_m \cdot (1-e^2) \cdot arc1''}$$

$$X_A'' = \frac{3958{,}10 \cdot 0{,}0067226700\,22 \cdot \cos^2(-31^o\,30'\,29{,}65'') \cdot \cos 340^o\,49'\,47{,}09'' \cdot sen 340^o\,49'\,47{,}09''}{6384226{,}17 \cdot (1-0{,}0067226700\,22) \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$X_A'' = -0{,}195''$$

Ángulos observados en terreno

Estación	Punto visado	Angulo leído	X’’	Angulo corregido	Angulo reducido
C	B	00º 00’ 00’’	0,219’’	00º 00’ 0,22’’	00º 00’ 00’’
	A	94º 36’ 42,00’’	-0,195’’	94º 36’ 41,80’’	94º 36’ 41,58’’

$$\varphi_B = -31^o\,18'\,8{,}32'' \rightarrow \varphi_A = -31^o\,30'\,29{,}65''\,;H_A = 3958{,}10m$$
$$\alpha_{BA} = \alpha_{AB} - 180^o \pm \Delta\alpha = 198^o\,22'\,44{,}67'' - 180^o - 0^o\,2'\,29{,}46'' \;\therefore\; \alpha_{BA} = 18^o\,20'\,15{,}21''$$
$$\varphi_m = \frac{\varphi_B + \varphi_A}{2} = -31^o\,24'\,18{,}98''$$
$$N_m = \frac{a}{(1-e^2 \cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1-0.0067226700\,22 \cdot (sen-31^o\,24'\,18{,}98'')^2)^{1/2}} = 6384219{,}62m$$

$$X_A'' = \frac{H_A \cdot e^2 \cdot \cos^2 \varphi_A \cdot \cos\alpha_{BA} \cdot sen\alpha_{BA}}{N_m \cdot (1-e^2) \cdot arc1''}$$

$$X_A'' = \frac{3958{,}10 \cdot 0{,}0067226700\,22 \cdot \cos^2(-31^o\,30'\,29{,}65'') \cdot \cos 18^o\,20'15{,}21'' \cdot sen 18^o\,20'15{,}21''}{638421{,}962 \cdot (1-0{,}0067226700\,22) \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$X_A'' = 0{,}188''$$

$$\varphi_B = -31^o\,18'\,8{,}32'' \rightarrow \varphi_C = -31^o\,21'\,21{,}33''\,;\, H_C = 3764{,}24m$$

$$\alpha_{BC} = \alpha_{BA} + \angle B \Rightarrow \alpha_{BC} = 18^o\,20'15{,}21'' + 47^o\,48'\,24{,}52'' \quad \therefore \alpha_{BC} = 66^o\,8'\,39{,}73''$$

$$\varphi_m = \frac{\varphi_B + \varphi_C}{2} = -31^o\,19'\,44{,}825''$$

$$N_m = \frac{a}{(1-e^2 \cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1-0.0067226700\,22 \cdot (sen -31^o\,19'\,44{,}825'')^2)^{1/2}} = 638419{,}22m$$

$$X_C'' = \frac{H_C \cdot e^2 \cdot \cos^2 \varphi_C \cdot \cos\alpha_{BC} \cdot sen\alpha_{BC}}{N_m \cdot (1-e^2) \cdot arc1''}$$

$$X_C'' = \frac{3764{,}24 \cdot 0{,}0067226700\,22 \cdot \cos^2(-31^o\,21'\,21{,}33'') \cdot \cos 66^o\,8'\,39{,}73'' \cdot sen 66^o\,8'\,39{,}73''}{638419{,}22 \cdot (1-0{,}0067226700\,22) \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$X_C'' = 0{,}222''$$

Ángulos observados en terreno

Estación	Punto visado	Angulo leído	X’’	Angulo corregido	Angulo reducido
B	A	00º 00’ 00’’	0,188’’	00º 00’ 0,19’’	00º 00’ 00’’
	C	47º 48’ 24,00’’	0,222’’	47º 48’ 24,22’’	47º 48’ 24,03’’

Estación	Punto visado	Angulo leído	X’’	Angulo corregido	Angulo reducido
A	C	00º 00’ 00’’	-0,186’’	359º 59’ 59,81’’	00º 00’ 00’’
	B	37º 34’ 53,1’’	0,178’’	37º 34’ 53,28’’	37º 34’ 53,47’’
B	A	00º 00’ 00’’	0,188’’	00º 00’ 0,19’’	00º 00’ 00’’
	C	47º 48’ 24,00’’	0,222’’	47º 48’ 24,22’’	47º 48’ 24,03’’
C	B	00º 00’ 00’’	0,219’’	00º 00’ 0,22’’	00º 00’ 00’’
	A	94º 36’ 42,00’’	-0,195’’	94º 36’ 41,80’’	94º 36’ 41,58’’

Ángulos interiores reducidos

$$\angle A: 37^\circ\, 34'\, 53{,}47''$$
$$\angle B: 47^\circ\, 48'\, 24{,}03''$$
$$\angle C: \underline{94^\circ\, 36'\, 41{,}58''}$$
$$\Sigma = 179^\circ\, 59'\, 59{,}08''$$

$$Error = 0^\circ\, 0'\, 0{,}92'' \Rightarrow Error\ unitario = \frac{0{,}92''}{3} = 0{,}30\bar{6}''$$

$$\text{I orden} \begin{cases} 1''\cdot \text{estación} \\ 2''\cdot\sqrt{n} \to 2''\cdot\sqrt{3} = 3{,}46''\text{; con "n" numero de estaciones.} \end{cases}$$

El error de cierre angular se encuentra dentro la tolerancia por lo tanto se puede compensar.

$\angle A: 37^\circ\, 34'\, 53{,}47''$	$\to 0{,}30\bar{6}''$	$\angle A: 37^\circ\, 34'\, 53{,}78''$		$\angle A: 37^\circ\, 34'\, 53{,}78''$
$\angle B: 47^\circ\, 48'\, 24{,}03''$	$\to 0{,}30\bar{6}''$	$\angle B: 47^\circ\, 48'\, 24{,}34''$		$\angle B: 47^\circ\, 48'\, 24{,}34''$
$\angle C: \underline{94^\circ\, 36'\, 41{,}58''}$	$\to 0{,}30\bar{6}''$	$\angle C: \underline{94^\circ\, 36'\, 41{,}89''}$	$\to -0{,}01''$	$\angle C: \underline{94^\circ\, 36'\, 41{,}88''}$
$\Sigma = 179^\circ\, 59'\, 59{,}08''$		$\Sigma = 180^\circ\, 0'\, 0{,}01''$		$\Sigma = 180^\circ$

Cierre angular de la poligonal quedando los ángulos compensados y esféricos.

$$E_U'' = \frac{0{,}67''}{3} = +0{,}22\bar{3}''$$

$\angle A: 37^\circ\, 34'\, 53{,}78''$	$\to +0{,}22\bar{3}$	$\angle A: 37^\circ\, 34'\, 54{,}00''$	$\to +0{,}01''$	$\angle A: 37^\circ\, 34'\, 54{,}01''$
$\angle B: 47^\circ\, 48'\, 24{,}34''$	$\to +0{,}22\bar{3}$	$\angle B: 47^\circ\, 48'\, 24{,}56''$		$\angle B: 47^\circ\, 48'\, 24{,}56''$
$\angle C: \underline{94^\circ\, 36'\, 41{,}88''}$	$\to +0{,}22\bar{3}$	$\angle C: \underline{94^\circ\, 36'\, 42{,}10''}$		$\angle C: \underline{94^\circ\, 36'\, 42{,}10''}$
$\Sigma = 180^\circ$		$\Sigma = 180^\circ\, 0'\, 0{,}66''$		$\Sigma = 180^\circ\, 0'\, 0{,}67''$

Calculo de alturas definitivas

<u>A</u>	<u>C</u>
$Z_1 = 90^\circ\, 41'\, 30{,}20''$	$Z_2 = 89^\circ\, 27'\, 16{,}00''$
$hi_1 = 1{,}40m$	$hi_2 = 1{,}46m$
$hb_2 = 1{,}25m$	$hb_1 = 0{,}00m$
$H_A = 3958{,}10m$	
$S = 17881{,}161m$	

Corrección de los ángulos cenitales:

$$\theta_1 = \frac{(hb_2 - hi_1)}{S \cdot arc1''} = \frac{(1{,}25 - 1{,}40)}{17881{,}161 \cdot 4{,}848136811 \cdot 10^{-6}} = -1{,}73''$$

$$\theta_2 = \frac{(hb_1 - hi_2)}{S \cdot arc1''} = \frac{(0{,}00 - 1{,}46)}{17881{,}161 \cdot 4{,}848136811 \cdot 10^{-6}} = -16{,}84''$$

$$Z_1c = Z_1 \pm \theta_1 = 90^o\ 41'\ 30{,}20'' - 1{,}73'' \Rightarrow Z_1c = 90^o\ 41'\ 28{,}47''$$

$$Z_2c = Z_2 \pm \theta_2 = 89^o\ 27'\ 16{,}00'' - 16{,}84'' \Rightarrow Z_2c = 89^o\ 26'\ 59{,}16''$$

$$\Delta h = S \cdot tg\,\frac{1}{2} \cdot (Z_2c - Z_1c) = 17881{,}161 \cdot tg\,\frac{1}{2} \cdot (89^o\ 26'\ 59{,}16'' - 90^o\ 41'\ 28{,}47'') = -193{,}73m$$

$$\varphi_m = \frac{\varphi_A + \varphi_C}{2} = -31^o\ 25'\ 55{,}49''$$

$$\rho_m = \frac{a \cdot (1 - e^2)}{(1 - e^2 \cdot sen^2\varphi_m)^{3/2}} = \frac{6378388 \cdot (1 - 0{,}006722670022)}{(1 - 0.006722670022 \cdot (sen - 31^o\ 25'\ 55{,}49'')^2)^{3/2}} = 6352922{,}089m$$

$$N_m = \frac{a}{(1 - e^2 \cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1 - 0.006722670022 \cdot (sen - 31^o\ 25'\ 55{,}49'')^2)^{1/2}} = 6384226{,}57m$$

$$\alpha_{AC} = \alpha_{AB} - \angle A = 198^o\ 22'\ 44{,}67'' - 37^o\ 34'\ 54{,}01'' \Rightarrow \alpha_{AC} = 160^o\ 47'\ 50{,}66''$$

$$R\alpha = \frac{N_m \cdot \rho_m}{\rho_m \cdot sen^2\alpha_{AC} + N_m \cdot \cos^2\alpha_{AC}}$$

$$= \frac{6384226{,}57 \cdot 6352922{,}089}{6352922{,}089 \cdot (sen160^o\ 47'\ 50.66'')^2 + 6384226{,}57 \cdot (\cos 160^o\ 47'\ 50.66'')^2}$$

$$R\alpha = 6356293{,}834m$$

$$A=\left(1+\frac{h_1}{R\alpha}\right)=\left(1+\frac{3958{,}10}{6356293{,}834}\right)=1{,}000622706$$

$$B=\left(1+\frac{|\Delta h|}{2\cdot R\alpha}\right)=\left(1+\frac{193{,}73}{2\cdot 6356293{,}834}\right)=1{,}000015239$$

$$C=\left(1+\frac{S^2}{12\cdot R\alpha^2}\right)=\left(1+\frac{17881{,}161^2}{12\cdot 6356293{,}934^2}\right)=1{,}000000659$$

$$\Delta h'=\Delta h\cdot A\cdot B\cdot C=-193{,}73\cdot 1{,}000622706\cdot 1{,}000015239\cdot 1{,}000000659=-193{,}85m$$

$$H_C=H_A\pm\Delta h'=3958{,}10-193{,}85\Rightarrow H_C=3764{,}25m$$

C	B
$Z_1=90^{\circ}\,13'\,51{,}00''$	$Z_2=89^{\circ}\,53'\,37{,}00''$
$hi_1=1{,}46m$	$hi_2=1{,}45m$
$hb_2=1{,}34m$	$hb_1=1{,}34m$
$H_C=3764{,}25m$	
$S=14719{,}998m$	

Corrección de los ángulos cenitales:

$$\theta_1=\frac{(hb_2-hi_1)}{S\cdot arc1''}=\frac{(1{,}34-1{,}46)}{14719{,}998\cdot 4{,}848136811\cdot 10^{-6}}=-1{,}68''$$

$$\theta_2=\frac{(hb_1-hi_2)}{S\cdot arc1''}=\frac{(1{,}34-1{,}45)}{14719{,}998\cdot 4{,}848136811\cdot 10^{-6}}=-1{,}54''$$

$$Z_1c=Z_1\pm\theta_1=90^{\circ}\,13'\,51{,}00''-1{,}68''\Rightarrow Z_1c=90^{\circ}\,13'\,49{,}32''$$

$$Z_2c=Z_2\pm\theta_2=89^{\circ}\,53'\,37{,}00''-1{,}54''\Rightarrow Z_2c=89^{\circ}\,53'\,35{,}46''$$

$$\Delta h=S\cdot tg\frac{1}{2}\cdot(Z_2c-Z_1c)=14719{,}998\cdot tg\frac{1}{2}\cdot\left(89^{\circ}\,53'\,35{,}46''-90^{\circ}\,13'\,49{,}32''\right)=-43{,}31m$$

$$\varphi_m=\frac{\varphi_C+\varphi_B}{2}=-31^{\circ}\,19'\,44{,}825''$$

$$\rho_m = \frac{a\cdot(1-e^2)}{(1-e^2\cdot sen^2\varphi_m)^{3/2}} = \frac{6378388\cdot(1-0{,}0067226700\!22)}{(1-0.0067226700\!22\cdot(sen-31^{o}\,19'\,44{,}825'')^2)^{3/2}} = 6352819{,}546m$$

$$N_m = \frac{a}{(1-e^2\cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1-0.0067226700\!22\cdot(sen-31^{o}\,19'\,44{,}825'')^2)^{1/2}} = 6384192{,}22m$$

$$\alpha_{CB} = \alpha_{CA} - \angle C = 340^{o}\,49'\,47{,}09'' - 94^{o}\,36'\,42{,}10'' \Rightarrow \alpha_{CB} = 246^{o}\,13'\,4.99''$$

$$R\alpha = \frac{N_m\cdot\rho_m}{\rho_m\cdot sen^2\alpha_{AC} + N_m\cdot\cos^2\alpha_{AC}}$$

$$= \frac{6384192{,}22\cdot 6352819{,}546}{6352819{,}546\cdot(sen\,246^{o}\,13'\,4.99'')^2 + 6384192{,}22\cdot(\cos 246^{o}\,13'\,4.99'')^2}$$

$$R\alpha = 6379069{,}435m$$

$$A = \left(1+\frac{h_1}{R\alpha}\right) = \left(1+\frac{3764{,}25}{6379069{,}435}\right) = 1{,}000590094$$

$$B = \left(1+\frac{|\Delta h|}{2\cdot R\alpha}\right) = \left(1+\frac{43{,}31}{2\cdot 6379069{,}435}\right) = 1{,}000003395$$

$$C = \left(1+\frac{S^2}{12\cdot R\alpha^2}\right) = \left(1+\frac{14719{,}998^2}{12\cdot 6379069{,}435^2}\right) = 1{,}000000444$$

$$\Delta h' = \Delta h\cdot A\cdot B\cdot C = -43{,}31\cdot 1{,}000590094\cdot 1{,}000003395\cdot 1{,}000000444 = -43{,}34m$$

$$H_B = H_C \pm \Delta h' = 3764{,}25 - 43{,}34 \Rightarrow H_B = 3720{,}91m$$

H_B fijo = 3720,89*m*
H_B calculado = 3720,91*m*
Error de cierre = -0,02*m*

Tolerancia para el error de cierre de una nivelación trigonométrica.

$$\mathrm{L} = \Sigma_{\text{DISTANCIA GEODÉSICA}} = 32601{,}159m = 32{,}601159Km.$$

$$\text{I orden}\begin{cases}0{,}1\cdot\sqrt{L}\rightarrow 0{,}1\cdot\sqrt{32{,}601159}=0{,}5709742464\approx 0{,}57m\\ 0{,}15\cdot\sqrt{L}\rightarrow 0{,}15\cdot\sqrt{32{,}601159}=0{,}8564613696\approx 0{,}86m\end{cases}$$

El error de cierre de altura se encuentra dentro la tolerancia por lo tanto se puede compensar.

El factor de compensación será: $FC=\dfrac{\text{error de cierre}}{L}$

$$FC=\frac{-0{,}02}{32601{,}159}=-6{,}134751222\cdot 10^{-7}$$

Compensación de las cotas:

$$C_n=FC\cdot\Sigma_S\ \text{recorrida}$$

$$C_C=FC\cdot 17881{,}161=-0{,}010964743\approx -0{,}01m$$

$$C_B=FC\cdot(17881{,}161+14719{,}998)=-0{,}02m$$

Por lo tanto la cota corregida seria:

$$H_{n\,corregida}=H_n\pm C_n$$

$$H_{C\,C}=3764{,}25-0{,}01\Rightarrow H_{C\,C}=3764{,}24m$$

$$H_{C\,B}=3720{,}91-0{,}02\Rightarrow H_{C\,B}=3720{,}89m$$

Calculo de distancias geodésicas definitivas

Calculo de distancia geodésica S_{AC}

$$\varphi_m=\frac{\varphi_A+\varphi_C}{2}=-31^{\circ}\,25'\,55{,}49''$$

$$\rho_m=\frac{a\cdot(1-e^2)}{(1-e^2\cdot sen^2\varphi_m)^{3/2}}=\frac{6378388\cdot(1-0{,}0067226700\!22)}{(1-0.0067226700\!22\cdot(sen-31^{\circ}\,25'\,55{,}49'')^2)^{3/2}}=6352922{,}089m$$

$$N_m=\frac{a}{(1-e^2\cdot sen^2\varphi_m)^{1/2}}=\frac{6378388}{(1-0.0067226700\!22\cdot(sen-31^{\circ}\,25'\,55{,}49'')^2)^{1/2}}=6384226{,}57m$$

$$\alpha_{AC}=\alpha_{AB}-\angle A=198^{\circ}\,22'\,44{,}67''-37^{\circ}\,34'\,54{,}01''\Rightarrow\alpha_{AC}=160^{\circ}\,47'\,50{,}66''$$

$$R\alpha = \frac{N_m \cdot \rho_m}{\rho_m \cdot sen^2\alpha_{AC} + N_m \cdot \cos^2 \alpha_{AC}}$$

$$= \frac{6384226{,}57 \cdot 6352922{,}089}{6352922{,}089 \cdot (sen160^o\ 47'\ 50{,}66'')^2 + 6384226{,}57 \cdot (\cos 160^o\ 47'\ 50{,}66'')^2}$$

$$R\alpha = 6356293{,}834m$$

$$Dh = \sqrt{Di^2 - \Delta h^2} = \sqrt{17893{,}074^2 - 193{,}86^2} = 17892{,}024m$$

$$hm = \frac{H_A + H_C}{2} = \frac{3958{,}10 + 3764{,}24}{2} = 3861{,}17m$$

$$Cnmm = \frac{-Dh \cdot hm}{R\alpha} = -\frac{17892{,}024 \cdot 3861{,}17}{6356293{,}834} = -10{,}869m$$

$$Dnmm = 17892{,}024 - 10{,}869 = 17881{,}155m$$

$$Cc = \frac{Dnmm^3}{24 \cdot R\alpha^2} = \frac{17881{,}155^3}{24 \cdot 6356293{.}834^2} = 0{,}006m$$

$$S = 17881{,}155 + 0{,}006 \Rightarrow S = 17881{,}161m$$

Calculo de distancia geodésica S_{CB}

$$\varphi_m = \frac{\varphi_C + \varphi_B}{2} = -31^o\ 19'\ 44{,}825''$$

$$\rho_m = \frac{a \cdot (1 - e^2)}{(1 - e^2 \cdot sen^2\varphi_m)^{3/2}} = \frac{6378388 \cdot (1 - 0{,}0067226700\,22)}{(1 - 0.0067226700\,22 \cdot (sen - 31^o\ 19'\ 44{,}825'')^2)^{3/2}} = 6352819{,}546m$$

$$N_m = \frac{a}{(1 - e^2 \cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1 - 0.0067226700\,22 \cdot (sen - 31^o\ 19'\ 44{,}825'')^2)^{1/2}} = 6384192{,}22m$$

$$\alpha_{CB} = \alpha_{CA} - \angle C = 340^o\ 49'\ 47{,}09'' - 94^o\ 36'\ 42{,}10'' \Rightarrow \alpha_{CB} = 246^o\ 13'\ 4.99''$$

$$R\alpha = \frac{N_m \cdot \rho_m}{\rho_m \cdot sen^2\alpha_{AC} + N_m \cdot \cos^2 \alpha_{AC}}$$

$$= \frac{6384192{,}22 \cdot 6352819{,}546}{6352819{,}546 \cdot \left(sen\, 246^{\circ}\, 13'\, 4.99''\right)^2 + 6384192{,}22 \cdot \left(\cos 246^{\circ}\, 13'\, 4.99''\right)^2}$$

$$R\alpha = 6379069{,}435m$$

$$Dh = \sqrt{Di^2 - \Delta h^2} = \sqrt{14728{,}70^2 - 43{,}35^2} = 14728{,}636m$$

$$hm = \frac{H_B + H_C}{2} = \frac{3720{,}89 + 3764{,}24}{2} = 3742{,}565m$$

$$Cnmm = \frac{-Dh \cdot hm}{R\alpha} = -\frac{14728{,}636 \cdot 3742{,}565}{6379069{,}435} = -8{,}641m$$

$$Dnmm = 14728{,}636 - 8{,}641 = 14719{,}995m$$

$$Cc = \frac{Dnmm^3}{24 \cdot R\alpha^2} = \frac{14719{,}995^3}{24 \cdot 6379069.435^2} = 0{,}003m$$

$$S = 14719{,}995 + 0{,}003 \Rightarrow S = 14719{,}998m$$

Calculo de posición por el problema directo (definitivo) (A $\rightarrow$ C)

Calculo de latitud

$$-\Delta\varphi'' = \underbrace{\frac{S \cdot \cos\alpha_{AC}}{\rho_m \cdot arc1''}}_{I} + \underbrace{\frac{S^2 \cdot sen^2\alpha_{AC} \cdot tg\varphi_m}{2 \cdot N_1 \cdot \rho_m \cdot arc1''}}_{II} - \underbrace{\frac{S^3 \cdot sen^2\alpha_{AC} \cdot \cos\alpha_{AC} \cdot \left(1 + 3 \cdot tg^2\varphi_m\right)}{6 \cdot N_1{}^2 \cdot \rho_m \cdot arc1''}}_{III}$$

Iteración 1

$$\alpha_{AC} = \alpha_{AB} - \angle A = 198^{\circ}\, 22'\, 44{,}67'' - 37^{\circ}\, 34'\, 54{,}01'' \Rightarrow \alpha_{AC} = 160^{\circ}\, 47'\, 50{,}66''$$
$$S = 17881{,}161m$$
$$\varphi_{m1} = \varphi_A = -31^{\circ}\, 30'\, 29{,}65''$$

$$\rho_{m1} = \frac{a \cdot \left(1 - e^2\right)}{\left(1 - e^2 \cdot sen^2 \varphi_{m1}\right)^{3/2}} = \frac{6378388 \cdot \left(1 - 0,006722670022\right)}{\left(1 - 0.006722670022 \cdot \left(sen - 31^{\circ}\, 30'\, 29,65''\right)^2\right)^{3/2}} = 6352998,058m$$

$$N_1 = \frac{a}{\left(1 - e^2 \cdot sen^2 \varphi_{m1}\right)^{1/2}} = \frac{6378388}{\left(1 - 0.006722670022 \cdot \left(sen - 31^{\circ}\, 30'\, 29,65''\right)^2\right)^{1/2}} = 6384252,017m$$

$$I_{(1)} = \frac{S \cdot \cos \alpha_{AC}}{\rho_{m1} \cdot arc1''} = \frac{17881,161 \cdot \cos 160^{\circ}\, 47'\, 50.66''}{6352998,058 \cdot 4,848136811 \cdot 10^{-6}} = -548,2522052''$$

$$II_{(1)} = \frac{S^2 \cdot sen^2 \alpha_{AC} \cdot tg \varphi_{m1}}{2 \cdot N_1 \cdot \rho_{m1} \cdot arc1''} = \frac{17881,161^2 \cdot \left(sen\, 160^{\circ}\, 47'\, 50.66''\right)^2 \cdot tg - 31^{\circ}\, 30'\, 29,65''}{2 \cdot 6384252,017 \cdot 6352998,058 \cdot 4,848136811 \cdot 10^{-6}}$$

$$II_{(1)} = -0,05391502994''$$

$$III_{(1)} = \frac{S^3 \cdot sen^2 \alpha_{AC} \cdot \cos \alpha_{AC} \cdot \left(1 + 3 \cdot tg^2 \varphi_{m1}\right)}{6 \cdot N_1^{\,2} \cdot \rho_{m1} \cdot arc1''}$$

$$= \frac{17881,161^3 \cdot \left(sen\, 160^{\circ}\, 47'\, 50.66''\right)^2 \cdot \cos 160^{\circ}\, 47'\, 50.66'' \cdot \left(1 + 3 \cdot \left(tg - 31^{\circ}\, 30'\, 29,65''\right)^2\right)}{6 \cdot 6384252,017^2 \cdot 6352998,058 \cdot 4,848136811 \cdot 10^{-6}}$$

$$III_{(1)} = -0,0001649613426''$$

$$-\Delta\varphi'' = I_{(1)} + II_{(1)} - II_{(1)} = -548,252'' - 0,054'' + 0,000 = -548,306''$$

$$\therefore \Delta\varphi = 0^{\circ}\, 9'\, 8,31''$$

$$\varphi_C = \varphi_A + \Delta\varphi \Rightarrow \varphi_C = -31^{\circ}\, 30'\, 29,65'' + 0^{\circ}\, 9'\, 8,31'' \;\therefore\; \varphi_C = -31^{\circ}\, 21'\, 21,34''$$

Iteración 2

$$\varphi_{m2} = \frac{\varphi_A + \varphi_C}{2} = \frac{-31^{\circ}\, 30'\, 29,65'' - 31^{\circ}\, 21'\, 21,34''}{2} = -31^{\circ}\, 25'\, 55,50''$$

$$\rho_{m2} = \frac{a \cdot (1-e^2)}{(1-e^2 \cdot sen^2\varphi_{m2})^{3/2}} = \frac{6378388 \cdot (1-0,0067226700222)}{(1-0.0067226700222 \cdot (sen -31^\circ\ 25'\ 55,50'')^2)^{3/2}} = 6352922,092m$$

$$N_1 = 6384252,017m$$

$$I_{(2)} = \frac{S \cdot \cos\alpha_{AC}}{\rho_{m2} \cdot arc1''} = \frac{17881,161 \cdot \cos 160^\circ\ 47'\ 50.66''}{6352922,092 \cdot 4,848136811 \cdot 10^{-6}} = -548,258761''$$

$$II_{(2)} = \frac{S^2 \cdot sen^2\alpha_{AC} \cdot tg\varphi_{m2}}{2 \cdot N_1 \cdot \rho_{m2} \cdot arc1''} = \frac{17881,161^2 \cdot (sen\,160^\circ\ 47'\ 50.66'')^2 \cdot tg -31^\circ\ 25'\ 55,50''}{2 \cdot 6384252,017 \cdot 6352922,092 \cdot 4,848136811 \cdot 10^{-6}}$$

$$II_{(2)} = -0,05375497379''$$

$$III_{(2)} = \frac{S^3 \cdot sen^2\alpha_{AC} \cdot \cos\alpha_{AC} \cdot (1+3tg^2\varphi_{m2})}{6 \cdot N_1^2 \cdot \rho_{m2} \cdot arc1''}$$

$$= \frac{17881,161^3 \cdot (sen\,160^\circ\ 47'\ 50.66'')^2 \cdot \cos 160^\circ\ 47'\ 50.66'' \cdot (1+3 \cdot (tg -31^\circ\ 25'\ 55,50'')^2)}{6 \cdot 6384252,017^2 \cdot 6352922,092 \cdot 4,848136811 \cdot 10^{-6}}$$

$$III_{(2)} = -0,0001644429792''$$

$$-\Delta\varphi'' = I_{(2)} + II_{(2)} - III_{(2)} = -548,259'' - 0,054'' + 0,000 = -548,313''$$

$$\therefore \Delta\varphi = 0^\circ\ 9'\ 8,31''$$

$$\varphi_C = \varphi_A + \Delta\varphi \Rightarrow \varphi_C = -31^\circ\ 30'\ 29,65'' + 0^\circ\ 9'\ 8,31'' \ \therefore \varphi_C = -31^\circ\ 21'\ 21,34''$$

$$\varphi_{m3} = \frac{\varphi_A + \varphi_C}{2} = \frac{-31^\circ\ 30'\ 29,65'' - 31^\circ\ 21'\ 21,34''}{2} = -31^\circ\ 25'\ 55,50''$$

$\varphi_{m2} = \varphi_{m3}$; Por lo tanto se cumple la convergencia

Calculo de longitud

$$N_2 = \frac{a}{(1-e^2 \cdot sen^2\varphi_C)^{1/2}} = \frac{6378388}{(1-0.0067226700222 \cdot (sen -31^\circ\ 21'\ 21,34'')^2)^{1/2}} = 6384201,158m$$

$$-\Delta\lambda'' = \frac{S \cdot sen\alpha_{AC}}{N_2 \cdot \cos\varphi_C \cdot arc1''} = \frac{17881{,}161 \cdot sen160^{o}\,47'\,50.66''}{6384201{,}158 \cdot \cos - 31^{o}\,21'\,21{,}34'' \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$= 222{,}5140681'' \Rightarrow \Delta\lambda''' = -222{,}51'' \therefore \Delta\lambda = -0^{o}\,3'\,42{,}51''$$

Por lo tanto la longitud es:

$$\lambda_C = \lambda_A + \Delta\lambda = -70^{o}\,37'\,12{,}37'' - 0^{o}\,3'\,42{,}51'' \therefore \lambda_C = -70^{o}\,40'\,54{,}88''$$

Calculo de azimut inverso α_{CA}

$$\alpha_{CA} = \alpha_{AC} + 180^{o} \pm \Delta\alpha$$

$$\Delta\varphi = \varphi_A - \varphi_C = -0^{o}\,9'\,8{,}31'' \Rightarrow \frac{\Delta\varphi}{2} = -0^{o}\,4'\,34{,}155'$$

$$\Delta\lambda = \lambda_A - \lambda_C = 222{,}51''$$

$$-\Delta\alpha'' = \frac{\Delta\lambda'' \cdot sen\varphi_m}{\cos\frac{1}{2}\cdot\Delta\varphi} = \frac{222{,}51'' \cdot sen(-31^{o}\,25'\,55{,}50'')}{\cos(-0^{o}\,4'\,34{,}155'')} \Rightarrow -\Delta\alpha'' = -116{,}0362871''$$

$$\Delta\alpha'' = 116{,}04'' \Rightarrow \Delta\alpha = 0^{o}\,1'\,56{,}04''$$

$$\alpha_{CA} = \alpha_{AC} + 180^{o} \pm \Delta\alpha = 160^{o}\,47'\,50{,}66'' + 180^{o} + 0^{o}\,1'\,56{,}04'' \Rightarrow \alpha_{CA} = 340^{o}\,49'\,46{,}70''$$

Calculo de posición por el problema directo
(C → B)

Calculo de latitud

$$-\Delta\varphi'' = \underbrace{\frac{S \cdot \cos\alpha_{CB}}{\rho_m \cdot arc1''}}_{I} + \underbrace{\frac{S^2 \cdot sen^2\alpha_{CB} \cdot tg\varphi_m}{2 \cdot N_1 \cdot \rho_m \cdot arc1''}}_{II} - \underbrace{\frac{S^3 \cdot sen^2\alpha_{CB} \cdot \cos\alpha_{CB} \cdot (1 + 3 \cdot tg^2\varphi_m)}{6 \cdot N_1{}^2 \cdot \rho_m \cdot arc1''}}_{III}$$

Iteración 1

$$\alpha_{CB} = \alpha_{CA} - \angle C = 340^{o}\,49'\,46{,}70'' - 94^{o}\,36'\,42{,}10'' \Rightarrow \alpha_{CB} = 246^{o}\,13'\,4.60''$$

$$S = 14719{,}998m$$

$$\varphi_{m1} = \varphi_C = -31^{o}\,21'\,21{,}34''$$

$$\rho_{m1} = \frac{a \cdot (1-e^2)}{(1-e^2 \cdot sen^2\varphi_{m1})^{3/2}} = \frac{6378388 \cdot (1-0{,}0067226700\,22)}{(1-0.0067226700\,22 \cdot (sen -31^o\,21'\,21{,}34'')^2)^{3/2}} = 6352846{,}228m$$

$$N_1 = \frac{a}{(1-e^2 \cdot sen^2\varphi_{m1})^{1/2}} = \frac{6378388}{(1-0.0067226700\,22 \cdot (sen -31^o\,21'\,21{,}34'')^2)^{1/2}} = 6384201{,}158m$$

$$I_{(1)} = \frac{S \cdot \cos\alpha_{CB}}{\rho_{m1} \cdot arc1''} = \frac{14719{,}998 \cdot \cos 246^o\,13'\,4.60''}{6352846{,}228 \cdot 4{,}848136811 \cdot 10^{-6}} = -192{,}7295507''$$

$$II_{(1)} = \frac{S^2 \cdot sen^2\alpha_{CB} \cdot tg\varphi_{m1}}{2 \cdot N_1 \cdot \rho_{m1} \cdot arc1''} = \frac{14719{,}998^2 \cdot (sen\,246^o\,13'\,4.60'')^2 \cdot tg -31^o\,21'\,21{,}34''}{2 \cdot 6384201{,}158 \cdot 6352846{,}228 \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$II_{(1)} = -0{,}2811411739''$$

$$III_{(1)} = \frac{S^3 \cdot sen^2\alpha_{CB} \cdot \cos\alpha_{CB} \cdot (1+3 \cdot tg^2\varphi_{m1})}{6 \cdot N_1^{\,2} \cdot \rho_{m1} \cdot arc1''}$$

$$= \frac{14719{,}998^3 \cdot (sen\,246^o\,13'\,4{,}60'')^2 \cdot \cos 246^o\,13'\,4{,}60'' \cdot (1+3 \cdot (tg -31^o\,21'\,21{,}34'')^2)}{6 \cdot 6384201{,}158^2 \cdot 6352846{,}228 \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$III_{(1)} = -0{,}0003022805821''$$

$$-\Delta\varphi'' = I_{(1)} + II_{(1)} - II_{(1)} = -192{,}730'' - 0{,}281'' + 0{,}000 = -193{,}011''$$

$$\therefore \Delta\varphi = 0^o\,3'\,13{,}01''$$

$$\varphi_B = \varphi_C + \Delta\varphi \Rightarrow \varphi_B = -31^o\,21'\,21{,}34'' + 0^o\,3'\,13{,}01'' \;\therefore\; \varphi_B = -31^o\,18'\,8{,}33''$$

Iteración 2

$$\varphi_{m2} = \frac{\varphi_B + \varphi_C}{2} = \frac{-31^o\,18'\,8{,}33'' - 31^o\,21'\,21{,}34''}{2} = -31^o\,19'\,44{,}835'' \approx -31^o\,19'\,44{,}84''$$

$$\rho_{m2} = \frac{a \cdot \left(1 - e^2\right)}{\left(1 - e^2 \cdot sen^2 \varphi_{m2}\right)^{3/2}} = \frac{6378388 \cdot \left(1 - 0{,}0067226700 22\right)}{\left(1 - 0.0067226700 22 \cdot \left(sen - 31^o\, 19'\, 44{,}84''\right)^2\right)^{3/2}} = 6352819{,}551 m$$

$$N_1 = 6384201{,}158 m$$

$$I_{(2)} = \frac{S \cdot \cos \alpha_{CB}}{\rho_{m2} \cdot arc1''} = \frac{14719{,}998 \cdot \cos 246^o\, 13'\, 4{,}60''}{6352819{,}551 \cdot 4{,}848136811 \cdot 10^{-6}} = -192{,}73036''$$

$$II_{(2)} = \frac{S^2 \cdot sen^2 \alpha_{CB} \cdot tg \varphi_{m2}}{2 \cdot N_1 \cdot \rho_{m2} \cdot arc1''} = \frac{14719{,}998^2 \cdot \left(sen\ 246^o\, 13'\, 4{,}60''\right)^2 \cdot tg - 31^o\, 19'\, 44{,}84''}{2 \cdot 6384201{,}158 \cdot 6352819{,}551 \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$II_{(2)} = -0{,}280846435''$$

$$III_{(2)} = \frac{S^3 \cdot sen^2 \alpha_{CB} \cdot \cos \alpha_{CB} \cdot \left(1 + 3 \cdot tg^2 \varphi_{m2}\right)}{6 \cdot N_1^{\ 2} \cdot \rho_{m2} \cdot arc1''}$$

$$= \frac{14719{,}998^3 \cdot \left(sen\ 246^o\, 13'\, 4{,}60''\right)^2 \cdot \cos 246^o\, 13'\, 4{,}60'' \cdot \left(1 + 3 \cdot \left(tg - 31^o\, 19'\, 44{,}84''\right)^2\right)}{6 \cdot 6384201{,}158^2 \cdot 6352819{,}551 \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$III_{(2)} = -0{,}0003019467126''$$

$$-\Delta\varphi'' = I_{(2)} + II_{(2)} - III_{(2)} = -192{,}730'' - 0{,}281'' + 0{,}000 = -193{,}011''$$

$$\therefore \Delta\varphi = 0^o\, 3'\, 13{,}01''$$

$$\varphi_B = \varphi_C + \Delta\varphi \Rightarrow \varphi_B = -31^o\, 21'\, 21{,}34'' + 0^o\, 3'\, 13{,}01'' \ \therefore \varphi_B = -31^o\, 18'\, 8{,}33''$$

$$\varphi_{m3} = \frac{\varphi_B + \varphi_C}{2} = \frac{-31^o\, 18'\, 8{,}33'' - 31^o\, 21'\, 21{,}34''}{2} = -31^o\, 19'\, 44{,}835''$$

$\varphi_{m2} = \varphi_{m3}$; por lo tanto se cumple la convergencia.

Calculo de longitud

$$N_2 = \frac{a}{\left(1 - e^2 \cdot sen^2 \varphi_B\right)^{1/2}} = \frac{6378388}{\left(1 - 0.0067226700 22 \cdot \left(sen - 31^o\, 18'\, 8{,}33''\right)^2\right)^{1/2}} = 6384183{,}288 m$$

$$-\Delta\lambda'' = \frac{S \cdot sen\alpha_{CB}}{N_2 \cdot \cos\varphi_B \cdot arc1''} = \frac{14719{,}998 \cdot sen\, 246^{\circ}\, 13'\, 4{,}60''}{6384183{,}288 \cdot \cos - 31^{\circ}\, 18'\, 8{,}33'' \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$= -509{,}3413965'' \Rightarrow \Delta\lambda''' = 509{,}34'' \therefore \Delta\lambda = 0^{\circ}\, 8'\, 29{,}34''$$

Por lo tanto la longitud es: $\lambda_B = \lambda_C + \Delta\lambda = -70^{\circ}\, 40'\, 54{,}89'' + 0^{\circ}\, 8'\, 29{,}34'' \therefore \lambda_B = -70^{\circ}\, 32'\, 25{,}55''$

$$\varphi_{B\,FIJO} = -31^{\circ}\, 18'\, 8{,}32'' \qquad \lambda_{B\,FIJO} = -70^{\circ}\, 32'\, 25{,}55''$$

$$\varphi_{B\,CALCULADO} = \underline{-31^{\circ}\, 18'\, 8{,}33} \qquad \lambda_{B\,CALCULADO} = \underline{-70^{\circ}\, 32'\, 25{,}55''}$$

$$\Delta\varphi'' = 0{,}01'' \qquad \Delta\lambda'' = 0{,}00''$$

Error en posición = $\dfrac{\sqrt{\varepsilon\varphi^2 + \varepsilon\lambda^2}}{\Sigma s}$

Arco de paralelo = $N_m \cdot \cos\varphi_m \cdot arc\, 1''$

Arco de meridiano = $\rho_m \cdot arc\, 1''$

$$\varphi_A = -31^{\circ}\, 30'\, 29{,}65''$$

$$\varphi_B = -31^{\circ}\, 18'\, 8{,}33''$$

$$\varphi_C = \underline{-31^{\circ}\, 21'\, 21{,}34''}$$

$$\varphi_m = -31^{\circ}\, 23'\, 19{,}77$$

$$\rho_m = \frac{a \cdot (1 - e^2)}{(1 - e^2 \cdot sen^2\varphi_m)^{3/2}} = \frac{6378388 \cdot (1 - 0{,}0067226700\!22)}{(1 - 0.0067226700\!22 \cdot (sen - 31^{\circ}\, 23'\, 19{,}77'')^2)^{3/2}} = 6352878{,}987m$$

$$N_m = \frac{a}{(1 - e^2 \cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1 - 0.0067226700\!22 \cdot (sen - 31^{\circ}\, 23'\, 19{,}77'')^2)^{1/2}} = 6384212{,}131m$$

Arco de paralelo = $N_m \cdot \cos\varphi_m \cdot arc\, 1'' = 6384212{,}131 \cdot \cos - 31^{\circ}\, 23'\, 19{,}77'' \cdot arc\, 1'' = 26{,}422m$

Arco de meridiano = $\rho_m \cdot arc\, 1'' = 6352878{,}987 \cdot arc\, 1'' = 30{,}800m$

$$\varepsilon\varphi = \Delta\varphi'' \cdot \text{Arco de meridiano} = 0{,}01'' \cdot 30{,}800 = 0{,}308$$

$$\varepsilon\lambda = \Delta\lambda'' \cdot \text{Arco de paralelo} = 0{,}00'' \cdot 26{,}422 = 0{,}00$$

Error lineal = $\sqrt{\varepsilon\varphi^2 + \varepsilon\lambda^2} = \sqrt{(0{,}308)^2 + (0)^2} = 0{,}308m$

$$\Sigma s = S_{AC} + S_{CB} = 17881{,}161 + 14719{,}998 = 32601{,}159m$$

Error de posición = $\frac{\text{Error lineal}}{\Sigma s} = \frac{0{,}308}{32601{,}159} = \frac{0{,}308 / 0{,}308}{32601{,}159 / 0{,}308} = \frac{1}{105847{,}9188}$

Error en posición = $\frac{\sqrt{\varepsilon\varphi^2 + \varepsilon\lambda^2}}{\Sigma s} \leq \frac{1}{40.000}$; para que el error de posición se encuentre dentro de la tolerancia de I orden.

Error en posición = $\frac{1}{105847{,}9188} < \frac{1}{40.000} \Rightarrow 0{,}0000094475116884 < 0{,}000025$

El error en posición se encuentra dentro de esta tolerancia (I orden) por lo tanto se puede compensar.

Factor de compensación; $FC_\varphi = \frac{\Delta\varphi''}{\Sigma S}, FC_\lambda = \frac{\Delta\lambda''}{\Sigma S}$

Corrección $n = FC_\varphi \cdot (S_1 + S_2 + ... + S_n)$ $\rightarrow$ corrección $n = FC_\lambda \cdot (S_1 + S_2 + ... + S_n)$

$FC_\varphi = \frac{0{,}01}{32601{,}159} = 3{,}067375611 \cdot 10^{-7}$; $FC_\lambda = \frac{-0{,}00}{32601{,}159} = 0$

$\text{Corrección } \varphi_C = -31^\circ\, 21'\, 21{,}34'' + FC_\varphi \cdot (17881{,}161) = -31^\circ\, 21'\, 21{,}34'' + 0{,}0055''$

$\varphi_C = -31^\circ\, 21'\, 21{,}33''$

$\text{Corrección } \varphi_B = -31^\circ\, 18'\, 8{,}33'' + FC_\varphi \cdot (17881{,}161 + 14719{,}998) = -31^\circ\, 18'\, 8{,}33'' + 0{,}01''$

$\varphi_B = -31^\circ\, 18'\, 8{,}32''$

$\text{Corrección } \lambda_C = -70^\circ\, 40'\, 54{,}88'' - FC_\lambda \cdot (17881{,}161) = -70^\circ\, 40'\, 54{,}88'' - 0{,}00''$

$\lambda_C = -70^\circ\, 40'\, 54{,}88''$

$\text{Corrección } \lambda_B = -70^\circ\, 32'\, 25{,}54'' - FC_\lambda \cdot (17881{,}161 + 14719{,}998) = -70^\circ\, 32'\, 25{,}54'' - 0{,}00''$

$\lambda_B = -70^\circ\, 32'\, 25{,}55''$

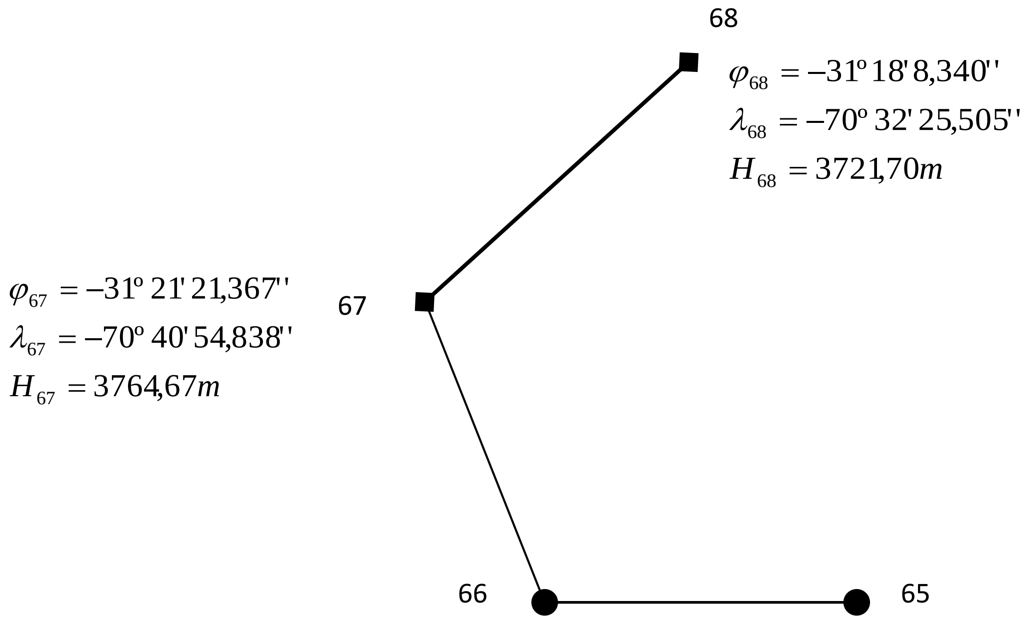
Por lo tanto las coordenadas geográficas C y su cota es:

$\varphi_C = -31^\circ\, 21'\, 21{,}33''$; $\lambda_C = -70^\circ 40' 54{,}88''$; $H_C = 3764{,}24m$

EJERCICIOS DE APLICACIÓN DE CALCULOS DE COORDENADAS GEOGRAFICAS Y COTAS

1) Calcular las coordenadas geográficas de 65 y su cota.

Elipsoide de referencia: SAD-69 Chua; $a = 6378388m$; $f = 1/298{,}25$



Estación	Punto	Angulo horizontal	Angulo vertical	Distancia inclinada	Altura de señal
67 hi = 1,46m	68 66	00º 00’ 00’’ 94º 36’ 41,59’’	------------- 89º 27’ 16’’	------------- 17893,074m	------------- 0,00m
66 hi = 1,40m	67 65	00º 00’ 00’’ 93º 30’ 03’’	90º 41’ 30,02’’ 87º 45’ 23,5’’	------------ 10667,978m	1,25m 2,00m
65 hi = 1,44m	66	-------------	92º 19’ 36,2’’	------------	2,00m

Solución:

Calculo de azimut inverso

$$\varphi_{68}: \quad -31^{o}\,18'\,8{,}340'' \qquad \lambda_{68}: \quad -70^{o}\,32'\,25{,}505''$$

$$\varphi_{67}: - -31^{o}\,21'\,21{,}367'' \qquad \lambda_{67}: - -70^{o}\,40'\,54{,}838''$$

$$\Delta\varphi = \quad 0^{o}\,3'\,13{,}027'' \qquad \Delta\lambda = \quad 0^{o}\,8'\,29{,}333''$$

$$\Delta\varphi'' = 193{,}027'' \qquad \Delta\lambda'' = 509{,}333''$$

$$\frac{\Delta\varphi''}{2} = 96{,}5135'' \qquad \frac{\Delta\lambda''}{2} = 254{,}6665''$$

$$\varphi_m = \frac{\varphi_{68} + \varphi_{67}}{2} = \frac{-31^{o}\,18'\,8{,}340'' - 31^{o}\,21'\,21{,}367''}{2} \Rightarrow \varphi_m = -31^{o}\,19'\,44{,}854''$$

$$arc1'' = 4{,}848136811 \cdot 10^{-6}$$

$$e^2 = 2 \cdot f - f^2 = 2 \cdot \frac{1}{298{,}25} - \left(\frac{1}{298{,}25}\right)^2 \Rightarrow e^2 = 0{,}0066945418\,55$$

$$\rho_m = \frac{a \cdot (1 - e^2)}{(1 - e^2 \cdot sen^2\varphi_m)^{3/2}} = \frac{6378160 \cdot (1 - 0{,}0066945418\,55)}{(1 - 0.0066945418\,55 \cdot (sen - 31^{o}\,19'\,44{,}854'')^2)^{3/2}} = 6352699{,}769m$$

$$N_m = \frac{a}{(1 - e^2 \cdot sen^2\varphi_m)^{1/2}} = \frac{6378160}{(1 - 0.0066945418\,55 \cdot (sen - 31^{o}\,19'\,44{,}854'')^2)^{1/2}} = 6383939{,}698m$$

$$S_1 \cdot sen\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = \Delta\lambda'' \cdot N_m \cdot \cos\varphi_m \cdot arc1'' \cdot \left[1 - \frac{(\Delta\lambda'' \cdot arc1'')^2}{24}\right]$$

$$= 509{,}333'' \cdot 6383939{,}698 \cdot \cos(-31^{o}\,19'\,44{,}854'') \cdot arc1'' \cdot \left[1 - \frac{(509{,}333'' \cdot arc1'')^2}{24}\right]$$

$$S_1 \cdot sen\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = 13465{,}49062$$

$$S_2 \cdot \cos\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = \Delta\varphi'' \cdot \rho_m \cdot \cos\frac{1}{2}\Delta\lambda \cdot arc1'' \cdot \left[1 - \frac{\rho_m{}^2 \cdot (\Delta\varphi'' \cdot arc1'')^2}{24 \cdot N_m{}^2}\right]$$

$$= 193{,}027'' \cdot 6352699{,}769 \cdot \cos(0^\circ\, 4'\, 14{,}6665'') \cdot arc1'' \cdot \left[1 - \frac{63532699{,}769^2 \cdot (193{,}027'' \cdot arc1'')^2}{24 \cdot 6383939{,}698^2}\right]$$

$$S_2 \cdot \cos\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = 5944{,}987037$$

Si $S_1 = S_2$; se tiene :

$$\frac{S_1 \cdot sen\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right)}{S_2 \cdot \cos\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right)} = tg\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = \frac{\Delta\lambda'' \cdot N_m \cdot \cos\varphi_m \cdot \left[1 - \frac{(\Delta\lambda'' \cdot arc1'')^2}{24}\right]}{\Delta\varphi'' \cdot \rho_m \cdot \cos\frac{1}{2}\Delta\lambda \cdot \left[1 - \frac{\rho_m{}^2 \cdot (\Delta\varphi'' \cdot arc1'')^2}{24 \cdot N_m{}^2}\right]}$$

$$tg\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = \frac{13465{,}49062}{5944{,}987037} = 2{,}265015977$$

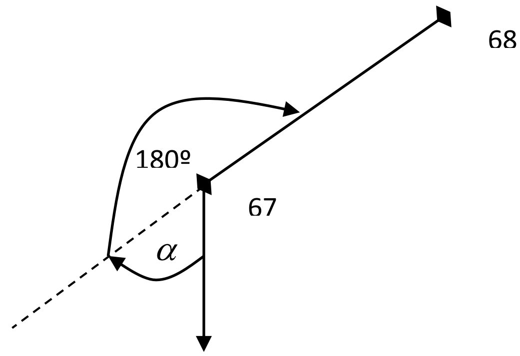
$$\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = tg^{-1}\, 2{,}265015977 = 66^\circ\, 10'\, 43{,}10''$$

$$-\Delta\alpha'' = \frac{\Delta\lambda'' \cdot sen\varphi_m}{\cos\frac{1}{2} \cdot \Delta\varphi} = \frac{509{,}333'' \cdot sen(-31^\circ\, 19'\, 44{,}854'')}{\cos(0^\circ\, 1'\, 36{,}5135'')} \Rightarrow -\Delta\alpha'' = -264{,}8295''$$

$$-\frac{\Delta\alpha''}{2} = -132{,}415'' \Rightarrow \frac{\Delta\alpha''}{2} = 132{,}415'' \quad \therefore \frac{\Delta\alpha}{2} = 0^\circ\, 2'\, 12{,}415''$$

$$\alpha = 66^\circ\, 10'\, 43{,}10'' + 0^\circ\, 2'\, 12{,}415'' \Rightarrow \alpha = 66^\circ\, 12'\, 55{,}515''$$

Por lo tanto $\alpha_{67-68} = \alpha + 180 = 66^\circ\, 12'\, 55{,}515'' + 180^\circ \Rightarrow \alpha_{67-68} = 246^\circ\, 12'\, 55{,}515''$



$$S_1 = \frac{|13465{,}49062|}{sen\left(\alpha + \frac{1}{2}\cdot\Delta\alpha\right)} \overset{SUR}{=} \frac{465{,}49062|}{sen(66^{o}\,10'\,43{,}10'')} \Rightarrow S_1 = 14719{,}453m$$

$$S_2 = \frac{|5944{,}987037|}{\cos\left(\alpha + \frac{1}{2}\cdot\Delta\alpha\right)} = \frac{|5944{,}987037|}{\cos(66^{o}\,10'\,43{,}10'')} \Rightarrow S_2 = 14719{,}453m$$

$S_1 = S_2$; Por lo tanto la distancia geodésica es: $S = 14719{,}453m$

Calculo de altura preliminar

67	66
$Z_1 = 89^{o}\,27'16''$	$Z_2 = 90^{o}41'\,30{,}2''$
$hi_1 = 1{,}46m$	$hi_2 = 1{,}40m$
$hb_2 = 0{,}00m$	$hb_1 = 1{,}25m$
$H_{67} = 3764{,}67m$	
$Di = 17893{,}074m$	

Corrección de los ángulos cenitales:

$$\theta_1 = \frac{(hb_2 - hi_1)\cdot senZ_1}{Di\cdot arc1''} = \frac{(0{,}00 - 1{,}46)\cdot sen\,89^{o}\,27'16''}{17893{,}074\cdot 4{,}848136811\cdot 10^{-6}} = -16{,}83''$$

$$\theta_2 = \frac{(hb_1 - hi_2)\cdot senZ_2}{Di\cdot arc1''} = \frac{(1{,}25 - 1{,}40)\cdot sen\,90^{o}\,41'\,30{,}2''}{17893{,}074\cdot 4{,}848136811\cdot 10^{-6}} = -1{,}73''$$

$$Z_1c = Z_1 \pm \theta_1 = 89^{o}\,27'16'' - 16{,}83'' \Rightarrow Z_1c = 89^{o}\,26'\,59{,}17''$$

$$Z_2c = Z_2 \pm \theta_2 = 90^{o}\,41'\,30{,}2'' - 1{,}73'' \Rightarrow Z_2c = 90^{o}\,41'\,28{,}47''$$

$$\Delta h' = Di\cdot sen\frac{1}{2}\cdot(Z_2c - Z_1c) = 17893{,}074\cdot sen\frac{1}{2}\cdot(90^{o}\,41'\,28{,}47'' - 89^{o}\,26'\,59{,}17'') = 193{,}85m$$

Por lo tanto $\Delta h' = 193{,}85m$

La cota preliminar de 66 es igual a: $H_{67} \pm \Delta h' = 3764{,}67 + 193{,}85 \Rightarrow H_{66} = 3958{,}52m$

Calculo de distancia geodésica preliminar

<u>67</u> <u>66</u>

$$\varphi_m = \varphi_{67} = -31^{o}\,21'\,21{,}367''$$

$$\rho_m = \frac{a \cdot (1 - e^2)}{(1 - e^2 \cdot sen^2 \varphi_m)^{3/2}} = \frac{6378160 \cdot (1 - 0{,}0066945418\,55)}{(1 - 0.0066945418\,55 \cdot (sen - 31^{o}\,21'\,21{,}367'')^2)^{3/2}} = 6352726{,}338m$$

$$N_m = \frac{a}{(1 - e^2 \cdot sen^2 \varphi_m)^{1/2}} = \frac{6378160}{(1 - 0.0066945418\,55 \cdot (sen - 31^{o}\,21'\,21{,}367'')^2)^{1/2}} = 6383948{,}598m$$

Sí $\alpha_{67-66} = \alpha_{67-68} + 94^{o}\,36'\,41{,}59'' \Rightarrow \alpha_{67-66} = 340^{o}\,49'\,37{,}105''$

$$R\alpha = \frac{N_m \cdot \rho_m}{\rho_m \cdot sen^2 \alpha_{67-66} + N_m \cdot \cos^2 \alpha_{67-66}}$$

$$= \frac{6383948{,}598 \cdot 6352726{,}338}{6352726{,}338 \cdot (sen 340^{o}\,49'\,37{,}105'')^2 + 6383948{,}598 \cdot (\cos 340^{o}\,49'\,37{,}105'')^2}$$

$$R\alpha = 6356079{,}301m$$

$$Dh = \sqrt{Di^2 - \Delta h^2} = \sqrt{17893{,}074^2 - 193{,}85^2} = 17892{,}024m$$

$$hm = \frac{H_{67} + H_{66}}{2} = \frac{3764{,}67 + 3958{,}52}{2} = 3861{,}595m$$

$$Cnmm = \frac{-Dh \cdot hm}{R\alpha} = -\frac{17892{,}024 \cdot 3861{,}595}{6356079{,}301} = -10{,}87m$$

$$Dnmm = 17892{,}024 - 10{,}87 = 17881{,}154m$$

$$Cc = \frac{Dnmm^3}{24 \cdot R\alpha^2} = \frac{17881{,}154^3}{24 \cdot 6356079{,}301^2} = 0{,}006m$$

$$S = Dnmm + Cc = 17881{,}154 + 0{,}006 \Rightarrow S = 17881{,}160m$$

Calculo de altura definitiva

<u>67</u>

$Z_1 = 89^o\ 27'\ 16''$
$hi_1 = 1{,}46m$
$hb_2 = 0{,}00m$
$H_{67} = 3764{,}67m$
$S = 17881{,}160m$

<u>66</u>

$Z_2 = 90^o 41'\ 30{,}2''$
$hi_2 = 1{,}40m$
$hb_1 = 1{,}25m$

Corrección de los ángulos cenitales:

$$\theta_1 = \frac{(hb_2 - hi_1)}{S \cdot arc1''} = \frac{(0{,}00 - 1{,}46)}{17881{,}160 \cdot 4{,}848136811 \cdot 10^{-6}} = -16{,}84''$$

$$\theta_2 = \frac{(hb_1 - hi_2)\cdot}{S \cdot arc1''} = \frac{(1{,}25 - 1{,}40)}{17881{,}160 \cdot 4{,}848136811 \cdot 10^{-6}} = -1{,}73''$$

$$Z_1c = Z_1 \pm \theta_1 = 89^o\ 27'\ 16'' - 16{,}84'' \Rightarrow Z_1c = 89^o\ 26'\ 59{,}16''$$

$$Z_2c = Z_2 \pm \theta_2 = 90^o\ 41'\ 30{,}2'' - 1{,}73'' \Rightarrow Z_2c = 90^o\ 41'\ 28{,}47''$$

$$\Delta h' = S \cdot tg\frac{1}{2} \cdot (Z_2c - Z_1c) = 17881{,}160 \cdot tg\frac{1}{2} \cdot (90^o\ 41'\ 28{,}47'' - 89^o\ 26'\ 59{,}16'') = 193{,}73m$$

Por lo tanto $\Delta h' = 193{,}73m$

$\alpha_{67-66} = 340^o\ 49'\ 37{,}105''$; $\rho_m = 6352726{,}338m$; $N_m = 6383948{,}598m$
$R\alpha = 6356079{,}301m$

$$A=\left(1+\frac{h_1}{R\alpha}\right)=\left(1+\frac{3764{,}67}{6356079{,}301}\right)=1{,}000592294$$

$$B=\left(1+\frac{|\Delta h'|}{2\cdot R\alpha}\right)=\left(1+\frac{193{,}73}{2\cdot 6356079{,}301}\right)=1{,}00001524$$

$$C=\left(1+\frac{S^2}{12\cdot R\alpha^2}\right)=\left(1+\frac{17881{,}160^2}{12\cdot 6356079{,}301^2}\right)=1{,}00000066$$

$$\Delta h=\Delta h'\cdot A\cdot B\cdot C=193{,}73\cdot 1{,}000592294\cdot 1{,}00001524\cdot 1{,}00000066=193{,}85m$$

$$H_{66}=H_{67}\pm\Delta h=3764{,}67+193{,}85\Rightarrow H_{66}=3958{,}52m$$

Calculo de distancia geodésica definitiva

<u>67</u> <u>66</u>

$$\varphi_m=\varphi_{67}=-31^{\circ}\,21'\,21{,}367''$$

$$\rho_m=\frac{a\cdot\left(1-e^2\right)}{\left(1-e^2\cdot sen^2\varphi_m\right)^{3/2}}=\frac{6378160\cdot\left(1-0{,}0066945418\,55\right)}{\left(1-0.0066945418\,55\cdot\left(sen-31^{\circ}\,21'\,21{,}367''\right)^2\right)^{3/2}}=6352726{,}338m$$

$$N_m=\frac{a}{\left(1-e^2\cdot sen^2\varphi_m\right)^{1/2}}=\frac{6378160}{\left(1-0.0066945418\,55\cdot\left(sen-31^{\circ}\,21'\,21{,}367''\right)^2\right)^{1/2}}=6383948{,}598m$$

Sí $\alpha_{67-66}=\alpha_{67-68}+94^{\circ}\,36'\,41{,}59''\Rightarrow\alpha_{67-66}=340^{\circ}\,49'\,37{,}105''$

$$R\alpha=\frac{N_m\cdot\rho_m}{\rho_m\cdot sen^2\alpha_{67-66}+N_m\cdot\cos^2\alpha_{67-66}}$$

$$=\frac{6383948{,}598\cdot 6352726{,}338}{6352726{,}338\cdot\left(sen340^{\circ}\,49'\,37{,}105''\right)^2+6383948{,}598\cdot\left(\cos 340^{\circ}\,49'\,37{,}105''\right)^2}$$

$$R\alpha=6356079{,}301m$$

$$Dh = \sqrt{Di^2 - \Delta h^2} = \sqrt{17893{,}074^2 - 193{,}85^2} = 17892{,}024m$$

$$hm = \frac{H_{67} + H_{66}}{2} = \frac{3764{,}67 + 3958{,}52}{2} = 3861{,}595m$$

$$Cnmm = \frac{-Dh \cdot hm}{R\alpha} = -\frac{17892{,}024 \cdot 3861{,}595}{6356079{,}301} = -10{,}87m$$

$$Dnmm = 17892{,}024 - 10{,}87 = 17881{,}154m$$

$$Cc = \frac{Dnmm^3}{24 \cdot R\alpha^2} = \frac{17881{,}154^3}{24 \cdot 6356079{,}301^2} = 0{,}006m$$

$$S = Dnmm + Cc = 17881{,}154 + 0{,}006 \Rightarrow S = 17881{,}160m$$

Calculo de posición por problema directo

(67 → 66)

$$-\Delta\varphi'' = \underbrace{\frac{S \cdot \cos\alpha_{67-66}}{\rho_m \cdot arc1''}}_{I} + \underbrace{\frac{S^2 \cdot sen^2\alpha_{67-66} \cdot tg\varphi_m}{2 \cdot N_1 \cdot \rho_m \cdot arc1''}}_{II} - \underbrace{\frac{S^3 \cdot sen^2\alpha_{67-66} \cdot \cos\alpha_{67-66} \cdot \left(1 + 3 \cdot tg^2\varphi_m\right)}{6 \cdot N_1{}^2 \cdot \rho_m \cdot arc1''}}_{III}$$

Iteración 1

$$\varphi_{m1} = \varphi_{67} = -31^{\circ}\,21'\,21{,}367''$$

$$S = 17881{,}160m$$

$$\alpha_{67-66} = 340^{\circ}\,49'\,37{,}105''$$

$$\rho_{m1} = \frac{a \cdot \left(1 - e^2\right)}{\left(1 - e^2 \cdot sen^2\varphi_{m1}\right)^{3/2}} = \frac{6378160 \cdot \left(1 - 0{,}0066945418\,55\right)}{\left(1 - 0.0066945418\,55 \cdot \left(sen - 31^{\circ}\,21'\,21{,}367''\right)^2\right)^{3/2}} = 6352726{,}338m$$

$$N_1 = \frac{a}{\left(1 - e^2 \cdot sen^2\varphi_{m1}\right)^{1/2}} = \frac{6378160}{\left(1 - 0.0066945418\,55 \cdot \left(sen - 31^{\circ}\,21'\,21{,}367''\right)^2\right)^{1/2}} = 6383948{,}598m$$

$$I_{(1)} = \frac{S \cdot \cos\alpha_{67-66}}{\rho_{m1} \cdot arc1''} = \frac{17881{,}160 \cdot \cos 340^{\circ}\,49'\,37{,}105''}{6352726{,}33\,8 \cdot 4{,}848136811 \cdot 10^{-6}} = 548{,}374097''$$

$$II_{(1)} = \frac{S^2 \cdot sen^2\alpha_{67-66} \cdot tg\varphi_{m1}}{2 \cdot N_1 \cdot \rho_{m1} \cdot arc1''} = \frac{17881{,}160^2 \cdot (sen\,340^{\circ}\,49'\,37{,}105'')^2 \cdot tg - 31^{\circ}\,21'\,21{,}367''}{2 \cdot 6383948{,}598 \cdot 6352726{,}338 \cdot 4{,}8481368111 \cdot 10^{-6}}$$

$$II_{(1)} = -0{,}05344''$$

$$III_{(1)} = \frac{S^3 \cdot sen^2\alpha_{67-66} \cdot \cos\alpha_{67-66} \cdot (1 + 3 \cdot tg^2\varphi_{m1})}{6 \cdot N_1^{\,2} \cdot \rho_{m1} \cdot arc1''}$$

$$= \frac{17881{,}160^3 \cdot (sen\,340^{\circ}\,49'\,37{,}105'')^2 \cdot \cos 340^{\circ}49'\,37{,}105'' \cdot (1 + 3 \cdot (tg - 31^{\circ}\,21'\,21{,}367'')^2)}{6 \cdot 6383948{,}598^2 \cdot 6352726{,}338 \cdot 4{,}8481368111 \cdot 10^{-6}}$$

$$III_{(1)} = 0{,}00016''$$

$$-\Delta\varphi'' = I_{(1)} + II_{(1)} - II_{(1)} = 548{,}374'' - 0{,}053'' - 0{,}0002 = 548{,}321''$$

$$\Delta\varphi'' = -548{,}321'' \Rightarrow \Delta\varphi = -0^{\circ}\,9'\,8{,}321''$$

$$\varphi_{66} = \varphi_{67} + \Delta\varphi \Rightarrow \varphi_{66} = -31^{\circ}\,21'\,21{,}367'' - 0^{\circ}\,9'\,8{,}321'' \;\therefore\; \varphi_{66} = -31^{\circ}\,30'\,29{,}688''$$

Iteración 2

$$\varphi_{m2} = \frac{\varphi_{67} + \varphi_{66}}{2} = \frac{-31^{\circ}\,21'\,21{,}367'' - 31^{\circ}30'29{,}688''}{2} = -31^{\circ}\,25'\,55{,}528''$$

$$\rho_{m2} = \frac{a \cdot (1 - e^2)}{(1 - e^2 \cdot sen^2\varphi_{m2})^{3/2}} = \frac{6378160 \cdot (1 - 0{,}00669454185 5)}{(1 - 0.0066945418 55 \cdot (sen - 31^{\circ}\,25'\,55{,}528'')^2)^{3/2}} = 6352801{,}883m$$

$$N_1 = 6383948{,}598m$$

$$I_{(2)} = \frac{S \cdot \cos\alpha_{67-66}}{\rho_{m2} \cdot arc1''} = \frac{17881{,}160 \cdot \cos 340^{\circ}\,49'\,37{,}105''}{6352801{,}883 \cdot 4{,}8481368111 \cdot 10^{-6}} = 548{,}367576''$$

$$II_{(2)} = \frac{S^2 \cdot sen^2\alpha_{67-66} \cdot tg\varphi_{m2}}{2 \cdot N_1 \cdot \rho_{m2} \cdot arc1''} = \frac{17881{,}160^2 \cdot (sen\,340^{\circ}\,49'\,37{,}105'')^2 \cdot tg - 31^{\circ}\,25'\,55{,}528''}{2 \cdot 6383948{,}598 \cdot 6352801{,}883 \cdot 4{,}8481368111 \cdot 10^{-6}}$$

$$II_{(2)} = -0{,}0535994''$$

$$III_{(2)} = \frac{S^3 \cdot sen^2\alpha_{67-66} \cdot \cos\alpha_{67-66} \cdot \left(1 + 3 \cdot tg^2\varphi_{m2}\right)}{6 \cdot N_1^{\ 2} \cdot \rho_{m2} \cdot arc1''}$$

$$= \frac{17881{,}160^3 \cdot (sen\,340^{o}\,49'\,37{,}105'')^2 \cdot \cos 340^{o}\,49'\,37{,}105'' \cdot \left(1 + 3 \cdot (tg - 31^{o}\,25'\,55{,}528'')^2\right)}{6 \cdot 6383948{,}598^2 \cdot 6352801{,}883 \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$III_{(2)} = 0{,}00016''$$

$$-\Delta\varphi'' = I_{(2)} + II_{(2)} - II_{(2)} = 548{,}368'' - 0{,}054'' - 0{,}0002'' = 548{,}3138''$$
$$\Delta\varphi'' = -548{,}314'' \Rightarrow \Delta\varphi = -0^{o}\,9'\,8{,}314''$$

$$\varphi_{66} = \varphi_{67} + \Delta\varphi \Rightarrow \varphi_{66} = -31^{o}\,21'\,21{,}367'' - 0^{o}\,9'\,8{,}314'' \therefore \varphi_{66} = -31^{o}\,30'\,29{,}681''$$

Iteración 3

$$\varphi_{m3} = \frac{\varphi_{67} + \varphi_{66}}{2} = \frac{-31^{o}\,21'\,21{,}367'' - 31^{o}\,30'\,29{,}681''}{2} \Rightarrow \varphi_{m3} = -31^{o}\,25'\,55.524''$$

$$\rho_{m3} = \frac{a \cdot \left(1 - e^2\right)}{\left(1 - e^2 \cdot sen^2\varphi_{m3}\right)^{3/2}} = \frac{6378160 \cdot (1 - 0{,}006694541855)}{\left(1 - 0.006694541855 \cdot (sen - 31^{o}\,25'\,55{,}524'')^2\right)^{3/2}} = 6352801{,}881 m$$

$$N_1 = 6383948{,}598 m$$
$$I_{(3)} = \frac{S \cdot \cos\alpha_{67-66}}{\rho_{m3} \cdot arc1''} = \frac{17881{,}160 \cdot \cos 340^{o}\,49'\,37{,}105''}{6352801{,}881 \cdot 4{,}848136811 \cdot 10^{-6}} = 548{,}3675762''$$

$$II_{(3)} = \frac{S^2 \cdot sen^2\alpha_{67-66} \cdot tg\varphi_{m3}}{2 \cdot N_1 \cdot \rho_{m3} \cdot arc1''} = \frac{17881{,}160^2 \cdot (sen\,340^{o}\,49'\,37{,}105'')^2 \cdot tg - 31^{o}\,25'\,55{,}524''}{2 \cdot 6383948{,}598 \cdot 6352801{,}881 \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$II_{(3)} = -0{,}0535994''$$
$$III_{(3)} = \frac{S^3 \cdot sen^2\alpha_{67-66} \cdot \cos\alpha_{67-66} \cdot \left(1 + 3 \cdot tg^2\varphi_{m3}\right)}{6 \cdot N_1^{\ 2} \cdot \rho_{m3} \cdot arc1''}$$

$$= \frac{17881{,}160^3 \cdot (sen\,340^{o}\,49'\,37{,}105'')^2 \cdot \cos 340^{o}\,49'\,37{,}105'' \cdot \left(1 + 3 \cdot (tg - 31^{o}\,25'\,55{,}524'')^2\right)}{6 \cdot 6383948{,}598^2 \cdot 6352801{,}881 \cdot 4{,}848136811 \cdot 10^{-6}}$$
$$III_{(3)} = 0{,}000164''$$

$$-\Delta\varphi'' = I_{(3)} + II_{(3)} - II_{(3)} = 548{,}368'' - 0{,}054'' - 0{,}0002'' = 548{,}3138''$$
$$\Delta\varphi'' = -548{,}314'' \Rightarrow \Delta\varphi = 0^{\circ}\,9'\,8{,}314''$$

$$\varphi_{66} = \varphi_{67} + \Delta\varphi \Rightarrow \varphi_{66} = -31^{\circ}\,21'\,21{,}367'' - 0^{\circ}\,9'\,8{,}314'' \;\therefore\; \varphi_{66} = -31^{\circ}\,30'\,29{,}681''$$

$$\varphi_{m4} = \frac{\varphi_{67} + \varphi_{66}}{2} = \frac{-31^{\circ}\,21'\,21{,}367'' - 31^{\circ}\,30'\,29{,}681''}{2} \Rightarrow \varphi_{m4} = -31^{\circ}\,25'\,55.524''$$

$\varphi_{m3} = \varphi_{m4}$; Por lo tanto se cumple la convergencia

Calculo de longitud

$$N_2 = \frac{a}{\left(1 - e^2 \cdot sen^2\varphi_{66}\right)^{1/2}} = \frac{6378160}{\left(1 - 0.0066945418 55 \cdot \left(sen - 31^{\circ}\,30'\,29{,}681''\right)^2\right)^{1/2}} = 6383999{,}242m$$

$$-\Delta\lambda'' = \frac{S \cdot sen\alpha_{67-66}}{N_2 \cdot \cos\varphi_{66} \cdot arc1''} = \frac{17881{,}160 \cdot sen340^{\circ}\,49'\,37{,}105''}{6383999{,}242 \cdot \cos - 31^{\circ}\,30'\,29{,}681'' \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$= -222{,}5526'' \Rightarrow \Delta\lambda'' = 222{,}553'' \Rightarrow \Delta\lambda = 0^{\circ}\,3'\,42{,}553''$$

Por lo tanto la longitud es:

$$\lambda_{66} = \lambda_{67} + \Delta\lambda = -70^{\circ}\,40'\,54{,}838'' + 0^{\circ}\,3'\,42{,}553'' \;\therefore\; \lambda_{66} = -70^{\circ}\,37'\,12{,}285''$$

Calculo de azimut inverso α_{66-67}

$$-\Delta\alpha'' = \frac{\Delta\lambda'' \cdot sen\varphi_m}{\cos\frac{\Delta\varphi}{2}} = \frac{222{,}553 \cdot sen - 31^{\circ}\,25'\,55{,}524''}{\cos 0^{\circ}\,4'\,34{,}157''} = 116{,}0590407''$$
$$\Delta\alpha'' = -116{,}06'' \;\therefore\; \Delta\alpha = -0^{\circ}\,1'\,56{,}06''$$

$$\alpha_{66-67} = \alpha_{67-66} - 180^{\circ} \pm \Delta\alpha = 340^{\circ}\,49'\,37{,}105'' - 180^{\circ} - 0^{\circ}\,1'\,56{,}06'' = 160^{\circ}\,47'\,41{,}045''$$

Por lo tanto $\alpha_{66-67} = 160^{\circ}\,47'\,41{,}045''$

Por lo tanto $\alpha_{66-65} = \alpha_{66-67} + 98^{\circ}\,30'\,3'' \Rightarrow \alpha_{66-65} = 259^{\circ}\,17'\,44{,}045''$

Calculo de altura preliminar

66	65
$Z_1 = 87^o\ 45'\ 23{,}5''$	$Z_2 = 92^o 19'\ 36{,}2''$
$hi_1 = 1{,}40m$	$hi_2 = 1{,}44m$
$hb_2 = 2{,}00m$	$hb_1 = 2{,}00m$
$H_{66} = 3958{,}52m$	
$Di = 10667{,}978m$	

Corrección de los ángulos cenitales:

$$\theta_1 = \frac{(hb_2 - hi_1) \cdot senZ_1}{Di \cdot arc1''} = \frac{(2{,}00 - 1{,}40) \cdot sen\, 87^o\ 45'\ 23{,}5''}{10667{,}978 \cdot 4{,}848136811 \cdot 10^{-6}} = 11{,}59''$$

$$\theta_2 = \frac{(hb_1 - hi_2) \cdot senZ_2}{Di \cdot arc1''} = \frac{(2{,}00 - 1{,}44) \cdot sen\, 92^o\ 19'\ 36{,}2''}{10667{,}978 \cdot 4{,}848136811 \cdot 10^{-6}} = 10{,}82''$$

$$Z_1c = Z_1 \pm \theta_1 = 87^o\ 45'\ 23{,}5'' + 11{,}59'' \Rightarrow Z_1c = 87^o\ 45'\ 35{,}09''$$

$$Z_2c = Z_2 \pm \theta_2 = 92^o\ 19'\ 36{,}2'' + 10{,}82'' \Rightarrow Z_2c = 92^o\ 19'\ 47{,}02''$$

$$\Delta h' = Di \cdot sen\, \frac{1}{2} \cdot (Z_2c - Z_1c) = 10667{,}978 \cdot sen\, \frac{1}{2} \cdot (92^o\ 19'\ 47{,}02'' - 87^o\ 45'\ 35{,}09'') = 425{,}33m$$

Por lo tanto $\Delta h' = 425{,}33m$

La cota preliminar de 65 es igual a: $H_{66} \pm \Delta h' = 3958{,}52 + 425{,}33 \Rightarrow H_{65} = 4383{,}85m$

Calculo de distancia geodésica preliminar

66 65

$$\varphi_m = \varphi_{66} = -31^o\ 30'\ 29{,}681''$$

$$\rho_m = \frac{a\cdot(1-e^2)}{(1-e^2\cdot sen^2\varphi_m)^{3/2}} = \frac{6378160\cdot(1-0{,}006694541855)}{(1-0.006694541855\cdot(sen-31^{o}\,30'\,29{,}681'')^2)^{3/2}} = 6352877{,}529m$$

$$N_m = \frac{a}{(1-e^2\cdot sen^2\varphi_m)^{1/2}} = \frac{6378160}{(1-0.006694541855\cdot(sen-31^{o}\,30'\,29{,}681'')^2)^{1/2}} = 6383999{,}242m$$

Sí $\alpha_{66-65} = 259^{o}\,17'\,44{,}045''$

$$R\alpha = \frac{N_m\cdot\rho_m}{\rho_m\cdot sen^2\alpha_{66-65} + N_m\cdot\cos^2\alpha_{66-65}}$$

$$= \frac{6383999{,}242\cdot 6352877{,}529}{6352877{,}529\cdot(sen\,259^{o}\,17'\,44{,}045'')^2 + 6383999{,}242\cdot(\cos 259^{o}\,17'\,44{,}045'')^2}$$

$$R\alpha = 6382920{,}455m$$

$$Dh = \sqrt{Di^2-\Delta h^2} = \sqrt{10667{,}978^2-425{,}33^2} = 10659{,}496m$$

$$hm = \frac{H_{66}+H_{65}}{2} = \frac{3958{,}52+4383{,}85}{2} = 4171{,}185m$$

$$Cnmm = \frac{-Dh\cdot hm}{R\alpha} = -\frac{10659{,}496\cdot 4171{,}185}{6382920{,}455} = -6{,}966m$$

$$Dnmm = 10659{,}496 - 10{,}87 = 10652{,}530m$$

$$Cc = \frac{Dnmm^3}{24\cdot R\alpha^2} = \frac{10652{,}530^3}{24\cdot 6382920{,}455^2} = 0{,}001m$$

$$S = Dnmm + Cc = 10652{,}530 + 0{,}001 \Rightarrow S = 10652{,}531m$$

Calculo de altura definitiva

66	65
$Z_1 = 87^o\, 45'\, 23{,}5''$	$Z_2 = 92^o 19'\, 36{,}2''$
$hi_1 = 1{,}40m$	$hi_2 = 1{,}44m$
$hb_2 = 2{,}00m$	$hb_1 = 2{,}00m$
$H_{66} = 3958{,}52m$	
$S = 10652{,}531m$	

Corrección de los ángulos cenitales:

$$\theta_1 = \frac{(hb_2 - hi_1)}{S \cdot arc1''} = \frac{(2{,}00 - 1{,}40)}{10652{,}531 \cdot 4{,}848136811 \cdot 10^{-6}} = 11{,}62''$$

$$\theta_2 = \frac{(hb_1 - hi_2)\cdot}{S \cdot arc1''} = \frac{(2{,}00 - 1{,}44)}{10652{,}531 \cdot 4{,}848136811 \cdot 10^{-6}} = 10{,}84''$$

$$Z_1c = Z_1 \pm \theta_1 = 87^o\, 45'\, 23{,}5'' + 11{,}62'' \Rightarrow Z_1c = 87^o\, 45'\, 35{,}12''$$

$$Z_2c = Z_2 \pm \theta_2 = 92^o\, 19'\, 36{,}2'' + 10{,}84'' \Rightarrow Z_2c = 92^o\, 19'\, 47{,}04''$$

$$\Delta h' = S \cdot tg\frac{1}{2} \cdot (Z_2c - Z_1c) = 10652{,}531 \cdot tg\frac{1}{2} \cdot (92^o\, 19'\, 47{,}04'' - 87^o\, 45'\, 35{,}12'') = 425{,}05m$$

Por lo tanto $\Delta h' = 425{,}05m$

$\alpha_{66-65} = 259^o\, 17'\, 44{,}045''$; $\rho_m = 6352877{,}529m$; $N_m = 6383999{,}242m$

$R\alpha = 6382920{,}455m$

$$A = \left(1 + \frac{h_1}{R\alpha}\right) = \left(1 + \frac{3958{,}52}{6382920{,}455}\right) = 1{,}000620174$$

$$B = \left(1 + \frac{|\Delta h'|}{2 \cdot R\alpha}\right) = \left(1 + \frac{425{,}05}{2 \cdot 6382920{,}455}\right) = 1{,}000033296$$

$$C = \left(1 + \frac{S^2}{12 \cdot R\alpha^2}\right) = \left(1 + \frac{10652{,}531^2}{12 \cdot 6382920{,}455^2}\right) = 1{,}000000232$$

$$\Delta h = \Delta h' \cdot A \cdot B \cdot C = 425{,}05 \cdot 1{,}000620174 \cdot 1{,}000033296 \cdot 1{,}000000232 = 425{,}33m$$

$$H_{65} = H_{66} \pm \Delta h = 3958{,}52 + 425{,}33 \Rightarrow H_{65} = 4383{,}85m$$

Calculo de distancia geodésica definitiva

66 65

$$\varphi_m = \varphi_{66} = -31^{\circ}\,30'\,29{,}681''$$

$$\rho_m = \frac{a\cdot(1-e^2)}{(1-e^2\cdot sen^2\varphi_m)^{3/2}} = \frac{6378160\cdot(1-0{,}0066945418555)}{(1-0.0066945418555\cdot(sen-31^{\circ}\,30'\,29{,}681'')^2)^{3/2}} = 6352877{,}529m$$

$$N_m = \frac{a}{(1-e^2\cdot sen^2\varphi_m)^{1/2}} = \frac{6378160}{(1-0.0066945418555\cdot(sen-31^{\circ}\,30'\,29{,}681'')^2)^{1/2}} = 6383999{,}242m$$

Sí $\alpha_{66-65} = 259^{\circ}\,17'\,44{,}045''$

$$R\alpha = \frac{N_m\cdot\rho_m}{\rho_m\cdot sen^2\alpha_{66-65} + N_m\cdot\cos^2\alpha_{66-65}}$$

$$= \frac{6383999{,}242\cdot 6352877{,}529}{6352877{,}529\cdot(sen\,259^{\circ}\,17'\,44{,}045'')^2 + 6383999{,}242\cdot(\cos 259^{\circ}\,17'\,44{,}045'')^2}$$

$$R\alpha = 6382920{,}455m$$

$$Dh = \sqrt{Di^2-\Delta h^2} = \sqrt{10667{,}978^2 - 425{,}33^2} = 10659{,}496m$$

$$hm = \frac{H_{66}+H_{65}}{2} = \frac{3958{,}52+4383{,}85}{2} = 4171{,}185m$$

$$Cnmm = \frac{-Dh\cdot hm}{R\alpha} = -\frac{10659{,}496\cdot 4171{,}185}{6382920.455} = -6{,}966m$$

$$Dnmm = 10659{,}496 - 6{,}966 = 10652{,}530m$$

$$Cc = \frac{Dnmm^3}{24\cdot R\alpha^2} = \frac{10652{,}530^3}{24\cdot 6382920{,}455^2} = 0{,}001m$$

$$S = Dnmm + Cc = 10652{,}530 + 0{,}001 \Rightarrow S = 10652{,}531m$$

Calculo de posición por problema directo

(66 → 65)

$$-\Delta\varphi'' = \underbrace{\frac{S\cdot\cos\alpha_{66-65}}{\rho_m\cdot arc1''}}_{I} + \underbrace{\frac{S^2\cdot sen^2\alpha_{66-65}\cdot tg\varphi_m}{2\cdot N_1\cdot\rho_m\cdot arc1''}}_{II} - \underbrace{\frac{S^3\cdot sen^2\alpha_{66-65}\cdot\cos\alpha_{66-65}\cdot\left(1+3\cdot tg^2\varphi_m\right)}{6\cdot N_1{}^2\cdot\rho_m\cdot arc1''}}_{III}$$

Iteración 1

$$\varphi_{m1} = \varphi_{66} = -31^{\circ}\,30'\,29{,}681''$$

$$S = 10652{,}531m$$

$$\alpha_{66-65} = 259^{\circ}\,17'\,44{,}045''$$

$$\rho_{m1} = \frac{a\cdot\left(1-e^2\right)}{\left(1-e^2\cdot sen^2\varphi_{m1}\right)^{3/2}} = \frac{6378160\cdot\left(1-0{,}0066945418\,55\right)}{\left(1-0.0066945418\,55\cdot\left(sen-31^{\circ}\,30'\,29{,}681''\right)^2\right)^{3/2}} = 6352877{,}529m$$

$$N_1 = \frac{a}{\left(1-e^2\cdot sen^2\varphi_{m1}\right)^{1/2}} = \frac{6378160}{\left(1-0.0066945418\,55\cdot\left(sen-31^{\circ}\,30'\,29{,}681''\right)^2\right)^{1/2}} = 6383999{,}242m$$

$$I_{(1)} = \frac{S\cdot\cos\alpha_{66-65}}{\rho_{m1}\cdot arc1''} = \frac{10652{,}531\cdot\cos 259^{\circ}\,17'\,44{,}045''}{6352877{,}529\cdot 4{,}848136811\cdot 10^{-6}} = -64{,}24199637'$$

$$II_{(1)} = \frac{S^2\cdot sen^2\alpha_{66-65}\cdot tg\varphi_{m1}}{2\cdot N_1\cdot\rho_{m1}\cdot arc1''} = \frac{10652{,}531^2\cdot\left(sen\,259^{\circ}\,17'\,44{,}045''\right)^2\cdot tg-31^{\circ}\,30'\,29{,}681''}{2\cdot 6383999{,}242\cdot 6352877{,}529\cdot 4{,}848136811\cdot 10^{-6}}$$

$$II_{(1)} = -0{,}1708962931'$$

$$III_{(1)} = \frac{S^3\cdot sen^2\alpha_{66-65}\cdot\cos\alpha_{66-65}\cdot\left(1+3\cdot tg^2\varphi_{m1}\right)}{6\cdot N_1{}^2\cdot\rho_{m1}\cdot arc1''}$$

$$= \frac{10652{,}531^3\cdot\left(sen\,259^{\circ}\,17'\,44{,}045''\right)^2\cdot\cos 259^{\circ}\,17'\,44{,}045''\cdot\left(1+3\cdot\left(tg-31^{\circ}\,30'\,29{,}681''\right)^2\right)}{6\cdot 6383999{,}242^2\cdot 6352877{,}529\cdot 4{,}848136811\cdot 10^{-6}}$$

$$III_{(1)} = -0{,}000061273'$$

$$-\Delta\varphi'' = I_{(1)} + II_{(1)} - II_{(1)} = -64{,}242'' - 0{,}171'' + 0{,}00006 = -64{,}413''$$
$$\Delta\varphi'' = 64{,}413'' \Rightarrow \Delta\varphi = 0^{o}\,1'\,4{,}413''$$
$$\varphi_{65} = \varphi_{66} + \Delta\varphi \Rightarrow \varphi_{65} = -31^{o}\,30'\,29{,}681'' + 0^{o}\,1'\,4{,}413'' \;\therefore\; \varphi_{65} = -31^{o}\,29'\,25{,}268''$$

Iteración 2

$$\varphi_{m2} = \frac{\varphi_{66} + \varphi_{65}}{2} = \frac{-31^{o}\,30'\,29{,}681'' - 31^{o}\,29'\,25{,}268''}{2} = -31^{o}\,29'\,57{,}475''$$

$$\rho_{m2} = \frac{a \cdot (1 - e^{2})}{(1 - e^{2} \cdot sen^{2}\varphi_{m2})^{3/2}} = \frac{6378160 \cdot (1 - 0{,}00669454185)}{(1 - 0.00669454185 \cdot (sen - 31^{o}\,29'\,57{,}475'')^{2})^{3/2}} = 6352868{,}637m$$

$$N_{1} = 6383999{,}242m$$

$$I_{(2)} = \frac{S \cdot \cos\alpha_{66-65}}{\rho_{m2} \cdot arc1''} = \frac{10652{,}531 \cdot \cos 259^{o}\,17'\,44{,}045''}{6352868{,}637 \cdot 4{,}848136811 \cdot 10^{-6}} = -64{,}24208629'$$

$$II_{(2)} = \frac{S^{2} \cdot sen^{2}\alpha_{66-65} \cdot tg\varphi_{m2}}{2 \cdot N_{1} \cdot \rho_{m2} \cdot arc1''} = \frac{10652{,}531^{2} \cdot (sen\,259^{o}\,17'\,44{,}045'')^{2} \cdot tg - 31^{o}\,29'\,57{,}475''}{2 \cdot 6383999{,}242 \cdot 6352868{,}637 \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$II_{(2)} = -0{,}170725174'$$

$$III_{(2)} = \frac{S^{3} \cdot sen^{2}\alpha_{66-65} \cdot \cos\alpha_{66-65} \cdot (1 + 3 \cdot tg^{2}\varphi_{m2})}{6 \cdot N_{1}^{2} \cdot \rho_{m2} \cdot arc1''}$$

$$= \frac{10652{,}531^{3} \cdot (sen\,259^{o}\,17'\,44{,}045'')^{2} \cdot \cos 259^{o}\,17'\,44{,}045'' \cdot (1 + 3 \cdot (tg - 31^{o}\,29'\,57{,}475'')^{2})}{6 \cdot 6383999{,}242^{2} \cdot 6352868{,}637 \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$III_{(2)} = -0{,}0000061273'$$

$$-\Delta\varphi'' = I_{(2)} + II_{(2)} - II_{(2)} = -64{,}242'' - 0{,}171'' - 0{,}0000612'' = -64{,}413''$$
$$\Delta\varphi'' = 64{,}413'' \Rightarrow \Delta\varphi = 0^{o}\,1'\,4{,}413''$$

$$\varphi_{65} = \varphi_{66} + \Delta\varphi \Rightarrow \varphi_{65} = -31^{o}\,30'\,29{,}681'' + 0^{o}\,1'\,4{,}413'' \;\therefore\; \varphi_{65} = -31^{o}\,29'\,25{,}268''$$
$$\varphi_{m3} = \frac{\varphi_{66} + \varphi_{65}}{2} = \frac{-31^{o}\,30'\,29{,}681'' - 31^{o}\,29'\,25{,}268''}{2} \Rightarrow \varphi_{m3} = -31^{o}\,29'\,57{,}475''$$

$\varphi_{m2} = \varphi_{m3}$; Por lo tanto se cumple la convergencia

Calculo de longitud

$$N_2 = \frac{a}{\left(1 - e^2 \cdot sen^2\varphi_{65}\right)^{1/2}} = \frac{6378160}{\left(1 - 0.006694541855 \cdot \left(sen - 31^o\,29'\,25{,}268''\right)^2\right)^{1/2}} = 6383993{,}286m$$

$$-\Delta\lambda'' = \frac{S \cdot sen\alpha_{66-65}}{N_2 \cdot \cos\varphi_{65} \cdot arc1''} = \frac{10652{,}531 \cdot sen259^o\,17'\,44{,}045''}{6383993{,}286 \cdot \cos - 31^o\,29'\,25{,}268'' \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$= -396{,}5984326'' \Rightarrow \Delta\lambda'' = 396{,}598'' \Rightarrow \Delta\lambda = 0^o\,6'\,36{,}598''$$

Por lo tanto la longitud es:

$\lambda_{65} = \lambda_{66} + \Delta\lambda = -70^o\,37'\,12{,}285'' + 0^o\,6'\,36{,}598'' \therefore \lambda_{65} = -70^o\,30'\,35{,}687''$

Por lo tanto las coordenadas geográficas de 65 y su cota respectiva es:

$\varphi_{65} = -31^o\,29'\,25{,}268''$

$\lambda_{65} = -70^o\,30'\,35{,}687''$

$H_{65} = 4383{,}85m$

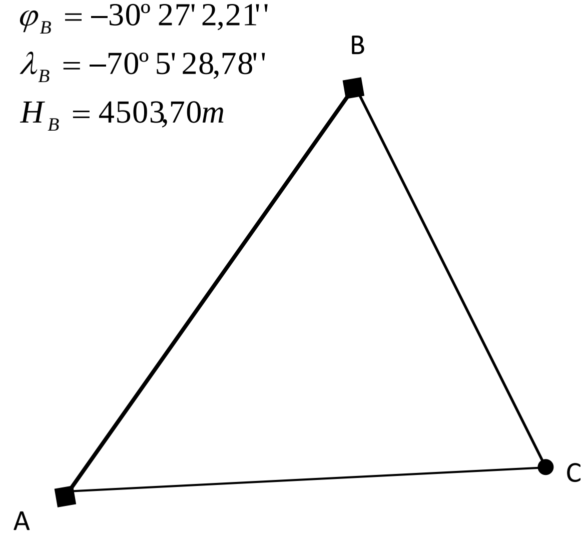
2) Calcular las coordenadas geograficas y cota del punto C, en el siguiente triangulo.

Estación	Punto visado	Angulo horizontal	Angulo vertical	Altura de la señal
A hi = 1,58m	C B	00º 00’ 00’’ 299º 31’ 57,50’’	89º 53’ 15,80’’ 89º 44’ 19,50’’	2,00m 2,00m
B hi = 1,56m	C A	00º 00’ 00’’ 31º 34’ 4,70’’	90º 16’ 59,80’’ 90º 19’ 3,40’’	2,00m 2,00m
C hi = 1,50m	A B	00º 00’ 00’’ 87º 57’ 51,30’’	90º 8’ 3,00’’ 89º 45’ 49,20’’	2,00m 2,00m

$\varphi_B = -30^{\circ}\, 27'\, 2{,}21''$

$\lambda_B = -70^{\circ}\, 5'\, 28{,}78''$

$H_B = 4503{,}70m$



$\varphi_A = -30^{\circ}\, 31'\, 10{,}38''$

$\lambda_A = -70^{\circ}\, 7'\, 9{,}77''$

$H_A = 4462{,}70m$

Sistema geodésico: PSAD-56

Elipsoide de referencia: Internacional 1924, $a = 6.378.388m$; $f = 1/297$

Nota: Las distancias calculadas son geodésicas.

Solución:

Cierre angular de la poligonal

$$\angle A: 60^{\circ}\,28'\,2{,}50''$$
$$\angle B: 31^{\circ}\,34'\,4{,}70''$$
$$\angle C: \underline{87^{\circ}\,57'\,51{,}30''}$$
$$\Sigma = 179^{\circ}\,59'\,58{,}50''$$

$$Error = +0^{\circ}\,0'\,1{,}50'' \Rightarrow Error\ unitario = +\frac{1{,}50''}{3} = +0{,}5''$$

$$\text{I orden} \begin{cases} 1''\cdot \text{estación} \\ 2''\cdot\sqrt{\text{n}} \rightarrow 2''\cdot\sqrt{3} = 3{,}46''\text{; con "n" numero de estaciones.} \end{cases}$$

El error de cierre angular se encuentra dentro la tolerancia por lo tanto se puede compensar.

$$\angle A: 60^{\circ}\,28'\,2{,}50'' \quad \rightarrow +0{,}5'' \quad \angle A: 60^{\circ}\,28'\,3{,}00''$$
$$\angle B: 31^{\circ}\,34'\,4{,}70'' \quad \rightarrow +0{,}5'' \quad \angle B: 31^{\circ}\,34'\,5{,}20''$$
$$\angle C: \underline{87^{\circ}\,57'\,51{,}30''} \quad \rightarrow +0{,}5'' \quad \angle C: \underline{87^{\circ}\,57'\,51{,}80''}$$
$$\Sigma = 179^{\circ}\,59'\,58{,}50'' \qquad\qquad \Sigma = 180^{\circ}$$

Calculo de la superficie de la poligonal

$$Sup = \sqrt{S\cdot(S-a)\cdot(S-b)\cdot(S-c)}\ \text{; con}\ S = \frac{(a+b+c)}{2}$$

Del Δ_{ABC} se tiene (por el teorema del seno):

$$\frac{S_{AB}}{senC} = \frac{S_{BC}}{senA} = \frac{S_{AC}}{senB}$$
$$S_{BC} = senA\cdot\frac{S_{AB}}{senC} = a$$
$$S_{AC} = senB\cdot\frac{S_{AB}}{senC} = b$$
$$S_{AB} = c$$

Calculo de S_{AB} y α_{AB}

$$\varphi_B : \quad -30^\circ\, 27'\, 2{,}21'' \qquad\qquad \lambda_B : \quad -70^\circ\, 5'\, 28{,}78''$$

$$\varphi_A : - -30^\circ\, 31'\, 10{,}38' \qquad\qquad \lambda_A : - -70^\circ\, 7'\, 9{,}77'$$

$$\Delta\varphi = \quad 0^\circ\, 4'\, 8{,}17'' \qquad\qquad \Delta\lambda = \quad 0^\circ\, 1'\, 40{,}99''$$

$$\Delta\varphi'' = 248{,}17'' \qquad\qquad \Delta\lambda'' = 100{,}99''$$

$$\frac{\Delta\varphi''}{2} = 124{,}085'' \qquad\qquad \frac{\Delta\lambda''}{2} = 50{,}495''$$

$$\varphi_m = \frac{\varphi_A + \varphi_B}{2} = \frac{-30^\circ\, 31'\, 10{,}38'' - 30^\circ\, 27'\, 2{,}21''}{2} \Rightarrow \varphi_m = -30^\circ\, 29'\, 6{,}295''$$

$$arc1'' = 4{,}848136811 \cdot 10^{-6}$$

$$e^2 = 2 \cdot f - f^2 = 2 \cdot \frac{1}{297} - \left(\frac{1}{297}\right)^2 \Rightarrow e^2 = 0{,}0067226700\,22$$

$$\rho_m = \frac{a \cdot \left(1 - e^2\right)}{\left(1 - e^2 \cdot sen^2\varphi_m\right)^{3/2}} = \frac{6378388 \cdot \left(1 - 0{,}0067226700\,22\right)}{\left(1 - 0.0067226700\,22 \cdot \left(sen - 30^\circ\, 29'\, 6{,}295''\right)^2\right)^{3/2}} = 6351986{,}349m$$

$$N_m = \frac{a}{\left(1 - e^2 \cdot sen^2\varphi_m\right)^{1/2}} = \frac{6378388}{\left(1 - 0.0067226700\,22 \cdot \left(sen - 30^\circ\, 29'\, 6{,}295''\right)^2\right)^{1/2}} = 6383913{,}104m$$

$$S_1 \cdot sen\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = \Delta\lambda'' \cdot N_m \cdot \cos\varphi_m \cdot arc1'' \cdot \left[1 - \frac{\left(\Delta\lambda'' \cdot arc1''\right)^2}{24}\right]$$

$$= 100{,}99'' \cdot 6383913{,}104 \cdot \cos\left(-30^\circ\, 29'\, 6{,}295''\right) \cdot arc1'' \cdot \left[1 - \frac{\left(100{,}99'' \cdot arc1''\right)^2}{24}\right]$$

$$S_1 \cdot sen\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = 2693{,}563247$$

$$S_2 \cdot \cos\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = \Delta\varphi'' \cdot \rho_m \cdot \cos\frac{1}{2}\Delta\lambda \cdot arc1'' \cdot \left[1 - \frac{\rho_m^{\ 2} \cdot \left(\Delta\varphi'' \cdot arc1''\right)^2}{24 \cdot N_m^{\ 2}}\right]$$

$$= 248{,}17'' \cdot 6351986{,}349 \cdot \cos\left(0^\circ\, 0'\, 50{,}495'\right) \cdot arc1'' \cdot \left[1 - \frac{6351986{,}349^2 \cdot \left(741{,}33'' \cdot arc1''\right)^2}{24 \cdot 6383913{,}104^2}\right]$$

$$S_2 \cdot \cos\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = 7642{,}468628$$

Si $S_1 = S_2$; se tiene :

$$\frac{S_1 \cdot sen\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right)}{S_2 \cdot \cos\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right)} = tg\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = \frac{\Delta\lambda'' \cdot N_m \cdot \cos\varphi_m \cdot \left[1 - \frac{(\Delta\lambda'' \cdot arc1'')^2}{24}\right]}{\Delta\varphi'' \cdot \rho_m \cdot \cos\frac{1}{2}\Delta\lambda \cdot \left[1 - \frac{\rho_m{}^2 \cdot (\Delta\varphi'' \cdot arc1'')^2}{24 \cdot N_m{}^2}\right]}$$

$$tg\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = \frac{2693{,}563247}{7642{,}468628} = 0{,}3524467523$$

$$\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right) = tg^{-1} 0{,}3524467523 = 19^{\circ}\ 24'\ 53{,}43''$$

$$-\Delta\alpha'' = \frac{\Delta\lambda'' \cdot sen\varphi_m}{\cos\frac{1}{2} \cdot \Delta\varphi} = \frac{100{,}99'' \cdot sen(-30^{\circ}\ 29'\ 6{,}295'')}{\cos(0^{\circ}\ 2'\ 4{,}085'')} \Rightarrow -\Delta\alpha'' = -51{,}23367835''$$

$$\Delta\alpha'' = 51{,}234'' \Rightarrow \frac{\Delta\alpha''}{2} = 25{,}62'' \Rightarrow \frac{\Delta\alpha}{2} = 0^{\circ}\ 0'\ 25{,}62''$$

$$\alpha = 19^{\circ}\ 24'\ 53{,}43'' + 0^{\circ}\ 0'\ 25{,}62'' \Rightarrow \alpha = 19^{\circ}\ 25'\ 19{,}05''$$

Por lo tanto $\alpha_{AB} = \alpha + 180^{\circ} \Rightarrow \alpha_{AB} = 199^{\circ}\ 25'\ 19{,}05''$

$$S_1 = \frac{|2693{,}563247|}{sen\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right)} = \frac{|2693{,}563247|}{sen(19^{\circ}\ 24'\ 53{,}43'')} \Rightarrow S_1 = 8103{,}246359m \approx 8103{,}25m$$

$$S_2 = \frac{|7642{,}468628|}{\cos\left(\alpha + \frac{1}{2} \cdot \Delta\alpha\right)} = \frac{|7642{,}468628|}{\cos(19^{\circ}\ 24'\ 53{,}43'')} \Rightarrow S_2 = 8103{,}246924m \approx 8103{,}25m$$

$\Delta S = 0{,}000565m$

$S_1 = S_2$; Por lo tanto la distancia geodésica es: $S_{AB} = 8103{,}25m$

$$\frac{S_{AB}}{senC} = \frac{S_{BC}}{senA} = \frac{S_{AC}}{senB}$$

$$S_{BC} = senA \cdot \frac{S_{AB}}{senC} = a \Rightarrow S_{BC} = sen\,60^{o}\,28'\,3{,}00'' \cdot \frac{8103{,}25}{sen\,87^{o}\,57'\,51{,}80''} = 7054{,}90m$$

$$S_{AC} = senB \cdot \frac{S_{AB}}{senC} = b \Rightarrow S_{AC} = sen\,31^{o}\,34'\,5{,}20'' \cdot \frac{8103{,}25}{sen\,87^{o}\,57'\,51{,}80''} = 4244{,}83m$$

$$S_{AB} = c \Rightarrow S_{AB} = 8103{,}25m$$

$$Sup = \sqrt{S \cdot (S-a) \cdot (S-b) \cdot (S-c)} \;;\text{con } S = \frac{(a+b+c)}{2}$$

$$S = \frac{(7054{,}90 + 4244{,}83 + 8103{,}25)}{2} = 9701{,}49m$$

$$Sup = \sqrt{9701{,}49 \cdot (9701{,}49 - 7054{,}90) \cdot (9701{,}49 - 4244{,}83) \cdot (9701{,}49 - 8103{,}25)}$$

$$Sup = 14963975{,}85m^{2}$$

Calculo de exceso esférico

$$\varphi_m = \frac{\varphi_A + \varphi_B}{2} = \frac{-30^{o}\,31'\,10{,}38'' - 30^{o}\,27'\,2{,}21''}{2} \Rightarrow \varphi_m = -30^{o}\,29'\,6{,}295''$$

$$\rho_m = \frac{a \cdot (1-e^2)}{(1 - e^2 \cdot sen^2 \varphi_m)^{3/2}} = \frac{6378388 \cdot (1 - 0{,}0067226700\,22)}{(1 - 0.0067226700\,22 \cdot (sen - 30^{o}\,29'\,6{,}295'')^2)^{3/2}} = 6351986{,}349m$$

$$N_m = \frac{a}{(1 - e^2 \cdot sen^2 \varphi_m)^{1/2}} = \frac{6378388}{(1 - 0.0067226700\,22 \cdot (sen - 30^{o}\,29'\,6{,}295'')^2)^{1/2}} = 6383913{,}104m$$

$$E'' = \frac{Superficie}{N_m \cdot \rho_m \cdot arc1''} = \frac{14963975{,}85}{6383913{,}104 \cdot 6351986{,}349 \cdot 4{,}848136811 \cdot 10^{-6}} = 0{,}0761159389'$$

$$E'' = 0{,}08'' \Rightarrow E_U'' = \frac{E''}{3} = \frac{0{,}08''}{3} \Rightarrow E_U'' = 0{,}02\overline{6}''$$

Cierre angular del polígono quedando los ángulos compensados y esféricos

$\angle A: 68^{o}\,28'\,3{,}00''$	$\rightarrow +0{,}02\overline{6}$	$\angle A: 68^{o}\,28'\,3{,}03''$		$\angle A: 68^{o}\,28'\,3{,}03''$
$\angle B: 31^{o}\,34'\,5{,}20''$	$\rightarrow +0{,}02\overline{6}$	$\angle B: 31^{o}\,34'\,5{,}23''$		$\angle B: 31^{o}\,34'\,5{,}23''$
$\angle C: 87^{o}\,57'\,51{,}80''$	$\rightarrow +0{,}02\overline{6}$	$\angle C: 87^{o}\,57'\,51{,}83''$	$\rightarrow -0{,}01''$	$\angle C: 87^{o}\,57'\,51{,}82''$
$\Sigma = 180^{o}$		$\Sigma = 180^{o}\,0'\,0{,}09''$		$\Sigma = 180^{o}\,0'\,0{,}08''$

Calculo de posición por el problema directo
(A $\rightarrow$ C)

Calculo de latitud

$$-\Delta\varphi''=\underbrace{\frac{S_{AC}\cdot\cos\alpha_{AC}}{\rho_m\cdot arc1''}}_{I}+\underbrace{\frac{S_{AC}{}^2\cdot sen^2\alpha_{AC}\cdot tg\varphi_m}{2\cdot N_1\cdot\rho_m\cdot arc1''}}_{II}-\underbrace{\frac{S_{AC}{}^3\cdot sen^2\alpha_{AC}\cdot\cos\alpha_{AC}\cdot\left(1+3\cdot tg^2\varphi_m\right)}{6\cdot N_1{}^2\cdot\rho_m\cdot arc1''}}_{III}$$

Iteración 1

$$\alpha_{AC}=\alpha_{AB}+\angle A=199^{\circ}\,25'19{,}05''+60^{\circ}\,28'3{,}03''\Rightarrow\alpha_{AC}=259^{\circ}\,53'\,22{,}08''$$

$$S_{AC}=4244{,}83m$$

$$\varphi_{m1}=\varphi_A=-30^{\circ}\,31'10{,}38''$$

$$\rho_{m1}=\frac{a\cdot\left(1-e^2\right)}{\left(1-e^2\cdot sen^2\varphi_{m1}\right)^{3/2}}=\frac{6378388\cdot\left(1-0{,}0067226700222\right)}{\left(1-0.0067226700222\cdot\left(sen-30^{\circ}\,31'10{,}38''\right)^2\right)^{3/2}}=6352020{,}111m$$

$$N_1=\frac{a}{\left(1-e^2\cdot sen^2\varphi_{m1}\right)^{1/2}}=\frac{6378388}{\left(1-0.0067226700222\cdot\left(sen-30^{\circ}\,31'10{,}38''\right)^2\right)^{1/2}}=6383924{,}414m$$

$$I_{(1)}=\frac{S_{AC}\cdot\cos\alpha_{AC}}{\rho_{m1}\cdot arc1''}=\frac{4244{,}83\cdot\cos 259^{\circ}\,53'\,22{,}08''}{6352020{,}111\cdot 4{,}848136811\cdot 10^{-6}}=-24{,}19740274'$$

$$II_{(1)}=\frac{S_{AC}{}^2\cdot sen^2\alpha_{AC}\cdot tg\varphi_{m1}}{2\cdot N_1\cdot\rho_{m1}\cdot arc1''}=\frac{4244{,}83^2\cdot\left(sen\ 259^{\circ}\,53'\,22{,}08''\right)^2\cdot tg-30^{\circ}\,31'10{,}38''}{2\cdot 6383924{,}414\cdot 6352020{,}111\cdot 4{,}848136811\cdot 10^{-6}}$$

$$II_{(1)}=-0{,}02618238529''$$

$$III_{(1)} = \frac{S_{AC}{}^{3} \cdot sen^{2}\alpha_{AC} \cdot \cos\alpha_{AC} \cdot \left(1 + 3 \cdot tg^{2}\varphi_{m1}\right)}{6 \cdot N_{1}{}^{2} \cdot \rho_{m1} \cdot arc1''}$$

$$= \frac{4244{,}83^{3} \cdot (sen\ 259^{o}\ 53'\ 22{,}08'')^{2} \cdot \cos 259^{o}\ 53'\ 22{,}08'' \cdot \left(1 + 3 \cdot (tg - 30^{o}\ 31'10{,}38'')^{2}\right)}{6 \cdot 6383924{,}414^{2} \cdot 6352020{,}111 \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$III_{(1)} = -0{,}000003529722546''$$

$$-\Delta\varphi'' = I_{(1)} + II_{(1)} - II_{(1)} = -24{,}197'' - 0{,}026'' + 0{,}000 = -24{,}223''$$

$$\therefore \Delta\varphi = 0^{o}\ 0'\ 24{,}22''$$

$$\varphi_{C} = \varphi_{A} + \Delta\varphi \Rightarrow \varphi_{C} = -30^{o}\ 31'10{,}38'' + 0^{o}\ 0'\ 24{,}22'' \ \therefore \varphi_{C} = -30^{o}\ 30'\ 46{,}16''$$

Iteración 2

$$\varphi_{m2} = \frac{\varphi_{A} + \varphi_{C}}{2} = \frac{-30^{o}\ 31'10{,}38'' - 30^{o}\ 30'\ 46{,}16''}{2} = -30^{o}\ 30'\ 58{,}27''$$

$$\rho_{m2} = \frac{a \cdot \left(1 - e^{2}\right)}{\left(1 - e^{2} \cdot sen^{2}\varphi_{m2}\right)^{3/2}} = \frac{6378388 \cdot (1 - 0{,}0067226700222)}{\left(1 - 0.0067226700222 \cdot (sen - 30^{o}\ 30'\ 58{,}27'')^{2}\right)^{3/2}} = 6352016{,}815m$$

$$N_{1} = 6383924{,}414m$$

$$I_{(2)} = \frac{S_{AC} \cdot \cos\alpha_{AC}}{\rho_{m2} \cdot arc1''} = \frac{4244{,}83 \cdot \cos 259^{o}\ 53'\ 22{,}08''}{6352016{,}815 \cdot 4{,}848136811 \cdot 10^{-6}} = -24{,}1974153''$$

$$II_{(2)} = \frac{S_{AC}{}^{2} \cdot sen^{2}\alpha_{AC} \cdot tg\varphi_{m2}}{2 \cdot N_{1} \cdot \rho_{m2} \cdot arc1''} = \frac{4244{,}83^{2} \cdot (sen\ 259^{o}\ 53'\ 22{,}08'')^{2} \cdot tg - 30^{o}\ 30'\ 58{,}27''}{2 \cdot 6383924{,}414 \cdot 6352016{,}815 \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$II_{(2)} = -0{,}0261788852''$$

$$III_{(2)} = \frac{S_{AC}^{3} \cdot sen^{2}\alpha_{AC} \cdot \cos\alpha_{AC} \cdot \left(1+3\cdot tg^{2}\varphi_{m2}\right)}{6\cdot N_{1}^{2}\cdot\rho_{m2}\cdot arc1''}$$

$$= \frac{4244{,}83^{3}\cdot\left(sen\ 259^{\circ}\ 53'\ 22{,}08''\right)^{2}\cdot\cos 259^{\circ}\ 53'\ 22{,}08''\cdot\left(1+3\cdot\left(tg-30^{\circ}\ 30'\ 58{,}27''\right)^{2}\right)}{6\cdot 6383924{,}414^{2}\cdot 6352016{,}815\cdot 4{,}848136811\cdot 10^{-6}}$$

$$III_{(2)} = -0{,}000003529240856''$$

$$-\Delta\varphi'' = I_{(2)} + II_{(2)} - III_{(2)} = -24{,}197''-0{,}026''+0{,}000 = -24{,}223''$$

$$\therefore \Delta\varphi = 0^{\circ}\ 0'\ 24{,}22''$$

$$\varphi_{C} = \varphi_{A} + \Delta\varphi \Rightarrow \varphi_{C} = -30^{\circ}\ 31'10{,}38''+0^{\circ}\ 0'\ 24{,}22''\ \therefore\ \varphi_{C} = -30^{\circ}\ 30'\ 46{,}16''$$

$$\varphi_{m3} = \frac{\varphi_{A}+\varphi_{C}}{2} = \frac{-30^{\circ}\ 31'10{,}38''-30^{\circ}\ 30'\ 46{,}16''}{2} = -30^{\circ}\ 30'\ 58{,}27''$$

$\varphi_{m2} = \varphi_{m3}$; Por lo tanto se cumple la convergencia

Calculo de longitud

$$N_{2} = \frac{a}{\left(1-e^{2}\cdot sen^{2}\varphi_{C}\right)^{1/2}} = \frac{6378388}{\left(1-0.006722670022\cdot\left(sen-30^{\circ}\ 30'\ 46{,}16''\right)^{2}\right)^{1/2}} = 6383922{,}206m$$

$$-\Delta\lambda'' = \frac{S_{AC}\cdot sen\alpha_{AC}}{N_{2}\cdot\cos\varphi_{C}\cdot arc1''} = \frac{4244{,}83\cdot sen\ 259^{\circ}\ 53'\ 22{,}08''}{6383922{,}206\cdot\cos-30^{\circ}\ 30'\ 46{,}16''\cdot 4{,}848136811\cdot 10^{-6}}$$

$$= -156{,}7247619'' \Rightarrow \Delta\lambda''' = 156{,}72''\ \therefore\ \Delta\lambda = 0^{\circ}\ 2'\ 36{,}72''$$

Por lo tanto la longitud es:

$$\lambda_{C} = \lambda_{A} + \Delta\lambda = -70^{\circ}\ 7'\ 9{,}77''+0^{\circ}\ 2'\ 36{,}72''\ \therefore\ \lambda_{C} = -70^{\circ}\ 4'\ 33{,}05''$$

Calculo de azimut inverso α_{CA}

$$\alpha_{CA} = \alpha_{AC} - 180^{\circ} \pm \Delta\alpha$$

$$\Delta\varphi = \varphi_A - \varphi_C = -0^\circ\,0'\,24{,}22'' \Rightarrow \frac{\Delta\varphi}{2} = -0^\circ\,0'\,12{,}11''$$

$$\Delta\lambda = \lambda_A - \lambda_C = -156{,}72''$$

$$-\Delta\alpha'' = \frac{\Delta\lambda''\cdot sen\varphi_m}{\cos\frac{1}{2}\cdot\Delta\varphi} = \frac{-156{,}72''\cdot sen(-30^\circ\,30'\,46{,}16'')}{\cos(-0^\circ\,0'\,12{,}11'')} \Rightarrow -\Delta\alpha'' = 79{,}57955699''$$

$$\Delta\alpha'' = -79{,}58'' \Rightarrow \Delta\alpha = -0^\circ\,1'\,19{,}58''$$

$$\alpha_{CA} = \alpha_{AC} - 180^\circ \pm \Delta\alpha = 259^\circ\,53'\,22{,}08'' - 180^\circ - 0^\circ\,1'\,19{,}58'' \Rightarrow \alpha_{CA} = 79^\circ\,52'\,2{,}50''$$

Calculo de posición por el problema directo
(C $\rightarrow$ B)

Calculo de latitud

$$-\Delta\varphi'' = \underbrace{\frac{S_{CB}\cdot\cos\alpha_{CB}}{\rho_m\cdot arc1''}}_{I} + \underbrace{\frac{S_{CB}{}^2\cdot sen^2\alpha_{CB}\cdot tg\varphi_m}{2\cdot N_1\cdot\rho_m\cdot arc1''}}_{II} - \underbrace{\frac{S_{CB}{}^3\cdot sen^2\alpha_{CB}\cdot\cos\alpha_{CB}\cdot(1+3\cdot tg^2\varphi_m)}{6\cdot N_1{}^2\cdot\rho_m\cdot arc1''}}_{III}$$

Iteración 1

$$\alpha_{CB} = \alpha_{CA} + \angle C = 79^\circ\,52'\,2{,}5'' + 87^\circ\,57'\,51{,}82'' \Rightarrow \alpha_{CB} = 167^\circ\,49'\,54{,}32''$$

$$S_{CB} = 7054{,}90m$$

$$\varphi_{m1} = \varphi_C = -30^\circ\,30'\,46{,}10''$$

$$\rho_{m1} = \frac{a\cdot(1-e^2)}{(1-e^2\cdot sen^2\varphi_{m1})^{3/2}} = \frac{6378388\cdot(1-0{,}0067226700\,22)}{(1-0.0067226700\,22\cdot(sen-30^\circ\,30'\,46{,}10'')^2)^{3/2}} = 6352013{,}519m$$

$$N_1 = \frac{a}{(1-e^2\cdot sen^2\varphi_{m1})^{1/2}} = \frac{6378388}{(1-0.0067226700\,22\cdot(sen-30^\circ\,30'\,46{,}10'')^2)^{1/2}} = 6383922{,}206m$$

$$I_{(1)} = \frac{S_{CB}\cdot\cos\alpha_{CB}}{\rho_{m1}\cdot arc1''} = \frac{7054{,}90\cdot\cos 167^\circ\,49'\,54{,}32''}{6352013{,}519\cdot 4{,}848136811\cdot 10^{-6}} = -223{,}9422053''$$

$$II_{(1)} = \frac{S_{CB}{}^{2} \cdot sen^{2}\alpha_{CB} \cdot tg\varphi_{m1}}{2 \cdot N_{1} \cdot \rho_{m1} \cdot arc1''} = \frac{7054{,}90^{2} \cdot (sen\,167^{o}\,49'\,54{,}32'')^{2} \cdot tg - 30^{o}\,30'\,46{,}16''}{2 \cdot 6383822{,}206 \cdot 6352013{,}519 \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$II_{(1)} = -0{,}0033145143448''$$

$$III_{(1)} = \frac{S_{CB}{}^{3} \cdot sen^{2}\alpha_{CB} \cdot \cos\alpha_{CB} \cdot (1 + 3 \cdot tg^{2}\varphi_{m1})}{6 \cdot N_{1}{}^{2} \cdot \rho_{m1} \cdot arc1''}$$

$$= \frac{7054{,}90^{3} \cdot (sen\,167^{o}\,49'\,54{,}32'')^{2} \cdot \cos 167^{o}\,49'\,54{,}32'' \cdot (1 + 3 \cdot (tg - 30^{o}\,30'\,46{,}16'')^{2})}{6 \cdot 6383922{,}206^{2} \cdot 6352013{,}519 \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$III_{(1)} = -0{,}00000413538309''$$

$$-\Delta\varphi'' = I_{(1)} + II_{(1)} - II_{(1)} = -223{,}942'' - 0{,}003'' + 0{,}000 = -233{,}945''$$

$$\therefore \Delta\varphi = 0^{o}\,3'\,53{,}95''$$

$$\varphi_{B} = \varphi_{C} + \Delta\varphi \Rightarrow \varphi_{B} = -30^{o}\,30'\,46{,}16'' + 0^{o}\,3'\,53{,}95'' \;\therefore\; \varphi_{B} = -30^{o}\,27'\,2{,}21''$$

Iteración 2

$$\varphi_{m2} = \frac{\varphi_{B} + \varphi_{C}}{2} = \frac{-30^{o}\,27'\,2{,}21'' - 30^{o}\,30'\,46{,}16''}{2} = -30^{o}\,28'\,54{,}19''$$

$$\rho_{m2} = \frac{a \cdot (1 - e^{2})}{(1 - e^{2} \cdot sen^{2}\varphi_{m2})^{3/2}} = \frac{6378388 \cdot (1 - 0{,}0067226700022)}{(1 - 0.0067226700022 \cdot (sen - 30^{o}\,28'\,54{,}19'')^{2})^{3/2}} = 6351983{,}056m$$

$$N_{1} = 6383922{,}206m$$

$$I_{(2)} = \frac{S_{CB} \cdot \cos\alpha_{CB}}{\rho_{m2} \cdot arc1''} = \frac{7054{,}90 \cdot \cos 167^{o}\,49'\,54{,}32''}{6351983{,}056 \cdot 4{,}848136811 \cdot 10^{-6}} = -223{,}9432793''$$

$$II_{(2)} = \frac{S_{CB}{}^{2} \cdot sen^{2}\alpha_{CB} \cdot tg\varphi_{m2}}{2 \cdot N_{1} \cdot \rho_{m2} \cdot arc1''} = \frac{7054{,}90^{2} \cdot (sen\,167^{o}\,49'\,54{,}32'')^{2} \cdot tg - 30^{o}\,28'\,54{,}19''}{2 \cdot 6383922{,}206 \cdot 6351983{,}056 \cdot 4{,}848136811 \cdot 10^{-6}}$$

$$II_{(2)} = -0{,}0033104181553''$$

$$III_{(2)} = \frac{S_{CB}{}^{3} \cdot sen^{2}\alpha_{CB} \cdot \cos\alpha_{CB} \cdot \left(1+3\cdot tg^{2}\varphi_{m2}\right)}{6\cdot N_{1}{}^{2} \cdot \rho_{m2} \cdot arc1''}$$

$$= \frac{7054{,}90^{3} \cdot (sen\,167^{o}\,49'\,54{,}32'')^{2} \cdot \cos 167^{o}\,49'\,54{,}32''\cdot\left(1+3\cdot(tg-30^{o}\,28'\,54{,}19'')^{2}\right)}{6\cdot 6383922{,}206^{2} \cdot 6351983{,}056 \cdot 4{,}8481368111\cdot 10^{-6}}$$

$$III_{(2)} = -0{,}000004130170189''$$

$$-\Delta\varphi'' = I_{(2)} + II_{(2)} - III_{(2)} = -223{,}943''-0{,}003''+0{,}000 = -223{,}946''$$

$$\therefore \Delta\varphi = 0^{o}\,3'\,53{,}95''$$

$$\varphi_{B} = \varphi_{C} + \Delta\varphi \Rightarrow \varphi_{B} = -30^{o}\,30'\,46{,}16''+0^{o}\,3'\,53{,}95'' \;\therefore\; \varphi_{B} = -30^{o}\,27'\,2{,}21''$$

$$\varphi_{m3} = \frac{\varphi_{B}+\varphi_{C}}{2} = \frac{-30^{o}\,27'\,2{,}21''-30^{o}\,30'\,46{,}16''}{2} = -30^{o}\,28'\,54{,}19''$$

$\varphi_{m2} = \varphi_{m3}$; Por lo tanto se cumple la convergencia.

Calculo de longitud

$$N_{2} = \frac{a}{\left(1-e^{2}\cdot sen^{2}\varphi_{B}\right)^{1/2}} = \frac{6378388}{\left(1-0.006722670022\cdot(sen-30^{o}\,27'\,2{,}21'')^{2}\right)^{1/2}} = 6383901{,}801m$$

$$-\Delta\lambda'' = \frac{S_{CB}\cdot sen\alpha_{CB}}{N_{2}\cdot\cos\varphi_{B}\cdot arc1''} = \frac{7054{,}90\cdot sen\,167^{o}\,49'\,54{,}32''}{6383901{,}801\cdot\cos-30^{o}\,27'\,2{,}21''\cdot 4{,}848136811\cdot 10^{-6}}$$

$$= 55{,}73458746'' \Rightarrow \Delta\lambda''' = -55{,}73'' \therefore \Delta\lambda = -0^{o}\,0'\,55{,}73''$$

Por lo tanto la longitud es:

$$\lambda_{B} = \lambda_{C} + \Delta\lambda = -70^{o}\,4'\,33{,}05''\text{ - }0^{o}\,0'\,55{,}73'' \therefore \lambda_{B} = -70^{o}\,5'\,28{,}78''$$

$$\begin{array}{ll} \varphi_{B\ FIJO} = -30^{o}\,27'\,2{,}21'' & \lambda_{B\ FIJO} = -70^{o}\,5'\,28{,}78'' \\ \varphi_{B\ CALCULADO} = \underline{-30^{o}\,27'\,2{,}21''} & \lambda_{B\ CALCULADO} = \underline{-70^{o}\,5'\,28{,}78''} \\ \Delta\varphi = 0 & \Delta\lambda = 0 \end{array}$$

Calculo de alturas

A

$Z_1 = 89^o\, 53'15{,}80''$
$hi_1 = 1{,}58m$
$hb_2 = 2{,}00m$
$H_A = 4462{,}70m$
$S = 4244{,}83m$

C

$Z_2 = 90^o\, 8'\, 3{,}00''$
$hi_2 = 1{,}50m$
$hb_1 = 2{,}00m$

Corrección de los ángulos cenitales:

$$\theta_1 = \frac{(hb_2 - hi_1)}{S \cdot arc1''} = \frac{(2{,}00 - 1{,}58)}{4244{,}83 \cdot 4{,}848136811 \cdot 10^{-6}} = 20{,}41''$$

$$\theta_2 = \frac{(hb_1 - hi_2)}{S \cdot arc1''} = \frac{(2{,}00 - 1{,}50)}{4244{,}83 \cdot 4{,}848136811 \cdot 10^{-6}} = 24{,}30''$$

$$Z_1c = Z_1 \pm \theta_1 = 89^o\, 53'15{,}80'' + 20{,}41'' \Rightarrow Z_1c = 89^o\, 53'\, 36{,}81''$$

$$Z_2c = Z_2 \pm \theta_2 = 90^o\, 8'\, 3{,}00'' + 24{,}30'' \Rightarrow Z_2c = 90^o\, 8'\, 27{,}30''$$

$$\Delta h = S \cdot tg\,\frac{1}{2} \cdot (Z_2c - Z_1c) = 4244{,}83 \cdot tg\,\frac{1}{2} \cdot (90^o\, 8'\, 27{,}30'' - 89^o\, 53'\, 36{,}81'') = 9{,}17m$$

$$\varphi_m = \frac{\varphi_A + \varphi_C}{2} = -30^o\, 30'\, 58{,}27''$$

$$\rho_m = \frac{a \cdot (1 - e^2)}{(1 - e^2 \cdot sen^2\varphi_m)^{3/2}} = \frac{6378388 \cdot (1 - 0{,}0067226700022)}{(1 - 0.0067226700022 \cdot (sen - 30^o\, 30'\, 58{,}27'')^2)^{3/2}} = 6352016{,}815m$$

$$N_m = \frac{a}{(1 - e^2 \cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1 - 0.0067226700022 \cdot (sen - 30^o\, 30'\, 58{,}27'')^2)^{1/2}} = 6383923{,}31m$$

$$\alpha_{AC} = \alpha_{AB} + \angle A = 199^o\, 25'19{,}05'' + 60^o\, 28'\, 3{,}03'' \Rightarrow \alpha_{AC} = 259^o\, 53'\, 22{,}08''$$

$$R\alpha = \frac{N_m \cdot \rho_m}{\rho_m \cdot sen^2\alpha_{AC} + N_m \cdot \cos^2\alpha_{AC}}$$

$$= \frac{6383923{,}31 \cdot 6352016{,}815}{6352016{,}815 \cdot (sen 259^{o}\, 53'\, 22{,}08'')^2 + 6383923{,}31 \cdot (\cos 259^{o}\, 53'\, 22{,}08'')^2}$$

$$R\alpha = 6382935{,}261 m$$

$$A = \left(1 + \frac{h_1}{R\alpha}\right) = \left(1 + \frac{4462{,}70}{6382935{,}261}\right) = 1{,}000699161$$

$$B = \left(1 + \frac{|\Delta h|}{2 \cdot R\alpha}\right) = \left(1 + \frac{9{,}17}{2 \cdot 6382935{,}261}\right) = 1{,}000000718$$

$$C = \left(1 + \frac{S^2}{12 \cdot R\alpha^2}\right) = \left(1 + \frac{4244{,}83^2}{12 \cdot 6382935{,}261^2}\right) = 1{,}000000037$$

$$\Delta h' = \Delta h \cdot A \cdot B \cdot C = 9{,}17 \cdot 1{,}00069916 \cdot 1{,}000000718 \cdot 1{,}000000037 = 9{,}18 m$$

$$H_C = H_A \pm \Delta h' = 4462{,}70 + 9{,}18 \Rightarrow H_C = 4471{,}88 m$$

C

$Z_1 = 89^{o}\, 45'\, 49{,}20''$

$hi_1 = 1{,}50 m$

$hb_2 = 2{,}00 m$

$H_C = 4471{,}88 m$

$S = 7054{,}90 m$

B

$Z_2 = 90^{o}\, 16'\, 59{,}80''$

$hi_2 = 1{,}56 m$

$hb_1 = 2{,}00 m$

Corrección de los ángulos cenitales:

$$\theta_1 = \frac{(hb_2 - hi_1)}{S \cdot arc 1''} = \frac{(2{,}00 - 1{,}50)}{7054{,}90 \cdot 4{,}848136811 \cdot 10^{-6}} = 14{,}62''$$

$$\theta_2 = \frac{(hb_1 - hi_2)}{S \cdot arc 1''} = \frac{(2{,}00 - 1{,}56)}{7054{,}90 \cdot 4{,}848136811 \cdot 10^{-6}} = 12{,}86''$$

$$Z_1c = Z_1 \pm \theta_1 = 89^\circ\,45'\,49{,}20''+14{,}62'' \Rightarrow Z_1c = 89^\circ\,46'\,3{,}82''$$

$$Z_2c = Z_2 \pm \theta_2 = 90^\circ\,16'\,59{,}80''+12{,}86'' \Rightarrow Z_2c = 90^\circ\,17'\,12{,}66''$$

$$\Delta h = S \cdot tg\,\frac{1}{2} \cdot (Z_2c - Z_1c) = 7054{,}90 \cdot tg\,\frac{1}{2} \cdot (90^\circ\,17'\,12{,}66'' - 89^\circ\,46'\,3{,}82'') = 31{,}98m$$

$$\varphi_m = \frac{\varphi_C + \varphi_B}{2} = -30^\circ\,30'\,46{,}16''$$

$$\rho_m = \frac{a \cdot (1-e^2)}{(1-e^2 \cdot sen^2\varphi_m)^{3/2}} = \frac{6378388 \cdot (1-0{,}0067226700\!22)}{(1-0.0067226700\!22 \cdot (sen-30^\circ\,30'\,46{,}16'')^2)^{3/2}} = 6352013{,}519m$$

$$N_m = \frac{a}{(1-e^2 \cdot sen^2\varphi_m)^{1/2}} = \frac{6378388}{(1-0.0067226700\!22 \cdot (sen-30^\circ\,30'\,46{,}16'')^2)^{1/2}} = 6383922{,}206m$$

$$\alpha_{CB} = \alpha_{CA} + \angle C = 79^\circ\,52'\,2{,}5''+87^\circ\,57'\,51{,}82'' \Rightarrow \alpha_{CB} = 167^\circ\,49'\,54{,}32''$$

$$R\alpha = \frac{N_m \cdot \rho_m}{\rho_m \cdot sen^2\alpha_{AC} + N_m \cdot \cos^2\alpha_{AC}}$$

$$= \frac{638392{,}206 \cdot 6352013{,}519}{6352013{,}519 \cdot (sen\,167^\circ\,49'\,54{,}32'')^2 + 6383922{,}206 \cdot (\cos 167^\circ\,49'\,54{,}32'')^2}$$

$$R\alpha = 6353424{,}433m$$

$$A = \left(1 + \frac{h_1}{R\alpha}\right) = \left(1 + \frac{4471{,}88}{6353424{,}433}\right) = 1{,}000703853$$

$$B = \left(1 + \frac{|\Delta h|}{2 \cdot R\alpha}\right) = \left(1 + \frac{31{,}96}{2 \cdot 6353424{,}433}\right) = 1{,}000002515$$

$$C = \left(1 + \frac{S^2}{12 \cdot R\alpha^2}\right) = \left(1 + \frac{7054{,}90^2}{12 \cdot 6353424{,}433^2}\right) = 1{,}000000103$$

$$\Delta h' = \Delta h \cdot A \cdot B \cdot C = 31{,}96 \cdot 1{,}000703853 \cdot 1{,}000002515 \cdot 1{,}000000103 = 31{,}98m$$

$$H_B = H_C \pm \Delta h' = 4471{,}88 + 31{,}98 \Rightarrow H_B = 4503{,}86m$$

H_B fijo = 4503,70*m*
H_B calculado = 4503,86*m*
Error de cierre = -0,16*m*

Tolerancia para el error de cierre de una nivelación trigonométrica.

$$\mathrm{L} = \Sigma_{\text{DISTANCIA GEODÉSICA}} = 11299{,}73m = 11{,}29973Km.$$

$$\text{I orden} \begin{cases} 0{,}1 \cdot \sqrt{\mathrm{L}} \rightarrow 0{,}1 \cdot \sqrt{11{,}29973} = 0{,}3361507102 \approx 0{,}34m \\ 0{,}15 \cdot \sqrt{\mathrm{L}} \rightarrow 0{,}15 \cdot \sqrt{11{,}29973} = 0{,}5042260653 \approx 0{,}50m \end{cases}$$

El error de cierre de altura se encuentra dentro la tolerancia por lo tanto se puede compensar.

El factor de compensación será: $FC = \dfrac{\text{error de cierre}}{\mathrm{L}}$

$$FC = \frac{-0{,}16}{11299{,}73} = -1{,}415963036 \cdot 10^{-5}$$

Compensación de las cotas:

$$C_n = FC \cdot \Sigma_S \text{ recorrida}$$
$$C_C = FC \cdot 4244{,}83 = -0{,}06010522374 \approx -0{,}06m$$
$$C_B = FC \cdot (4244{,}83 + 7054{,}90,) = -0{,}16m$$

Por lo tanto la cota corregida seria:

$$H_{n\,corregida} = H_n \pm C_n$$
$$H_{C\,C} = 4471{,}88 - 0{,}06 \Rightarrow H_{C\,C} = 4471{,}82m$$
$$H_{C\,B} = 4503{,}86 - 0{,}16 \Rightarrow H_{C\,B} = 4503{,}70m$$

Por lo tanto las coordenadas geográficas de C y su cota es:

$$\varphi_C = -30^{\circ}\, 30'\, 46{,}16''$$
$$\lambda_C = -70^{\circ}\, 4'\, 33{,}05''$$
$$H_C = 4471{,}82m$$